10-2020

# A Panel Data Model with Generalized Higher-Order Network Effects

Badi Baltagi
bbaltagi@maxwell.syr.edu

Sophia Ding
*ETH Zurich*, ding@kof.ethz.ch

Peter Egger
*ETH Zurich*, egger@kof.ethz.ch

CENTER FOR POLICY RESEARCH

THE MAXWELL SCHOOL

WORKING PAPER SERIES

# A Panel Data Model with Generalized Higher-Order Network Effects

Badi H. Baltagi, Sophia Ding, and Peter H. Egger

Paper No. 233
October 2020

ISSN: 1525-3066

Maxwell Syracuse University | Center for Policy Research
426 Eggers Hall
Syracuse University
Syracuse, NY 13244-1020
(315) 443-3114/ email: ctrpol@syr.edu
http://www.maxwell.syr.edu/CPR_Working_Papers.aspx

# CENTER FOR POLICY RESEARCH – Fall 2020

**Leonard M. Lopoo, Director**
**Professor of Public Administration and International Affairs (PAIA)**

**Associate Directors**

Margaret Austin
Associate Director, Budget and Administration

John Yinger
Trustee Professor of Economics (ECON) and Public Administration and International Affairs (PAIA)
Associate Director, Center for Policy Research

## SENIOR RESEARCH ASSOCIATES

Badi Baltagi, ECON
Robert Bifulco, PAIA
Leonard Burman, PAIA
Carmen Carrión-Flores, ECON
Alfonso Flores-Lagunes, ECON
Sarah Hamersma, PAIA
Madonna Harrington Meyer, SOC
Colleen Heflin, PAIA
William Horrace, ECON
Yilin Hou, PAIA
Hugo Jales, ECON
Jeffrey Kubik, ECON
Yoonseok Lee, ECON
Amy Lutz, SOC
Yingyi Ma, SOC
Katherine Michelmore, PAIA
Jerry Miner, ECON
Shannon Monnat, SOC
Jan Ondrich, ECON
David Popp, PAIA
Stuart Rosenthal, ECON
Michah Rothbart, PAIA
Alexander Rothenberg, ECON
Rebecca Schewe, SOC
Amy Ellen Schwartz, PAIA/ECON
Ying Shi, PAIA
Saba Siddiki, PAIA
Perry Singleton, ECON
Yulong Wang, ECON
Peter Wilcoxen, PAIA
Maria Zhu, ECON

## GRADUATE ASSOCIATES

Rhea Acuña, PAIA
Graham Ambrose, PAIA
Mariah Brennan, SOC. SCI.
Ziqiao Chen, PAIA
Yoon Jung Choi, PAIA
Dahae Choo, ECON
Stephanie Coffey, PAIA
William Clay Fannin, PAIA
Giuseppe Germinario, ECON
Myriam Gregoire-Zawilski, PAIA
Jeehee Han, PAIA
Mary Helander, SOC. SCI.
Amra Kandic, SOC
Sujung Lee, SOC
Mattie Mackenzie-Liu, PAIA
Maeve Maloney, ECON
Austin McNeill Brown, SOC. SCI.
Qasim Mehdi, PAIA
Nicholas Oesterling, PAIA
Claire Pendergrast, SOC
Lauryn Quick, PAIA
Krushna Ranaware, SOC
Sarah Reilly, SOC
Christopher Rick, PAIA
Spencer Shanholtz, PAIA
Sarah Souders, PAIA
Joaquin Urrego, ECON
Yao Wang, ECON
Yi Yang, ECON
Xiaoyan Zhang, Human Dev.
Bo Zheng, PAIA
Dongmei Zuo, SOC. SCI.

## STAFF

Joseph Boskovski, Manager, Maxwell X Lab
Ute Brady, Postdoctoral Scholar
Willy Chen, Research Associate
Katrina Fiacchi, Administrative Specialist
Michelle Kincaid, Senior Associate, Maxwell X Lab
Emily Minnoe, Administrative Assistant
Candi Patterson, Computer Consultant
Samantha Trajkovski, Postdoctoral Scholar
Laura Walsh, Administrative Assistant

## Abstract

Many data situations require the consideration of network effects among the cross-sectional units of observation. In this paper, we present a generalized panel model which accounts for two features: (i) three types of network effects on the right-hand side of the model, namely through weighted dependent variable, weighted exogenous variables, as well as weighted error components, and (ii) higher-order network effects due to ex-ante unknown network-decay functions or the presence of multiplex (or multi-layer) networks among all of those. We outline the model, the basic assumptions, and present simulation results.



**JEL No.:** C23, C33, C34

**Keywords:** Spatial and Network Interdependence, Panel Data, Higher-Order Network Effects



**Authors**: Badi H. Baltagi, Department of Economics and Center for Policy Research, Syracuse University, 426 Eggers Hall, Syracuse, NY 13244-1020, US, bbaltagi@maxwell.syr.edu; Sophia Ding, ETH Zurich, LEE G103, Leonhardstrasse 21, 8092 Zurich, Switzerland, ding@kof.ethz.ch.; Peter H. Egger, ETH Zurich and CEPR, LEE G101, Leonhardstrasse 21, 8092 Zurich, Switzerland, egger@kof.ethz.ch.

# 1 Introduction[1]

In the past two decades, we have seen a big surge in the development of models for the analysis of data which are characterized by some form of spatial or network interdependence between the cross-sectional units of observation both for cross-sectional and panel data (for examples of the latter see Baltagi and Pesaran, 2007; and Bai, Baltagi, and Pesaran, 2016). While this development roots in a more narrow interest on mere geographical or spatial connections between units (see for instance Pinkse, Slade, and Brett, 2002; Egger, Pfaffermayr, and Winner, 2005; Baltagi, Egger and Pfaffermayr, 2007; Ertur and Koch, 2007; Baltagi, Egger, and Kesina, 2017), more recent research widened its scope to cover other forms of interdependence through networks, covering inter alia the following: social interactions between individuals; trade or affiliate networks between firms; migration, commuting, and transport links between regions; to mention a few.

One discomfort with associated work had been that network links were parameterized in a way such that only a single unknown scalar would parameterize the intensity of network links apart from an otherwise fully known structure of the decay or intensity of those links (see Badinger and Egger, 2011, 2013, 2015; Gupta and Robinson, 2015).[2] An obvious way of parting with this admittedly restrictive approach is to allow for a richer parametrization of the network links through so-called higher-order network links. Suppose a network is characterized by the mere geography and spatial location of the cross-sectional units. Rather than assuming that the decay in space is fully known up to a multiplicative scalar (the so-called spatial or network lag parameter which is commonly estimated), one could estimate a separate parameter for different rings of neighbors or units in a specific quantile of the distance distribution. More specifically, consider tax competition

[1]This paper is written in honor of M. Hashem Pesaran for his many contributions to econometrics including panel data and cross-section dependence; see, e.g., the recent survey by Chudik and Pesaran (2015) in the Oxford Handbook of Panel Data. We would like to thank an anonymous referee and Alex Chudik for their useful comments. Egger gratefully acknowledges funding from SNF under grant no. 100018-169537.

[2]The mentioned papers consider various forms of higher-order processes. In the papers of Badinger and Egger, the number of network layers is finite, whereas in Gupta and Robinson it may grow asymptotically. Most of the mentioned work considers panel data with a random effects structure. However, what is different between this paper and the mentioned work is that the dimensions (number of layers) and the parameters governing the network structure in the time-invariant error component differ from the ones in the time-variant component. In doing so, the present work is a blend feature of processes studied, e.g., in Badinger and Egger, and the network-effects model in Baltagi, Egger, and Pfaffermayr (2013), which considers a single network layer with one parameter each in the time-invariant and time-variant error components.

between municipalities. One can allow for the estimation of a separate parameter measuring the effect of competitive pressure from the taxes set by immediate neighbors, by second-order (or indirect) neighbors, third-order neighbors, etc. This could be implemented by estimating a separate parameter measuring the effect of the competitive pressure from the taxes set by municipalities, say within a distance of 10 kilometers; between 10 and 20 kilometers; and so on. In this way, the data estimates the relatively flexible decay function, and much less is assumed about the decay structure of network links ex ante. Moreover, one can allow for different network channels or types of network links – in a so-called multiplex network. For example, geographical, cultural, and historical ties between countries could be considered simultaneously, and their relative importance could be estimated.

The present paper considers the estimation of a random-effects panel-data model where four types of higher-order network links can be present simultaneously: (i) one involving the dependent variable; (ii) another involving the exogenous explanatory variables; (iii) a third involving the cross-sectional (time-invariant) error component; and (iv) a fourth involving the remainder (time-variant) error component. Overall, we could dub this approach a high-order network-lag Durbin model with heterogeneous network processes regarding the error components.

The remainder of the paper is organized as follows. We introduce the notation and outline the econometric model as well as the key assumptions in the subsequent section. Section 3 presents a design and the associated results of a Monte Carlo simulation, which focuses on the bias, root-mean-squared-error, and likelihood-ratio-test findings in cases with relatively small data sets. The last section concludes with a brief summary.

# 2 Model

## 2.1 Notation and outline

Formally, the model can be introduced as

$$y = (I_T \otimes \tilde{L})y + X\beta + u \tag{1}$$

$$u = Z_\mu u_1 + u_2 \tag{2}$$

$$u_1 = \tilde{A}u_1 + \mu, \tag{3}$$

$$u_2 = (I_T \otimes \tilde{B})u_2 + \nu \tag{4}$$

where $y' = (y_{11}, ..., y_{1N}, ..., y_{T1}, ..., y_{TN})$ ordered such that the slow index is $t = 1, ..., T$ and the fast index is $i = 1, ..., N$. The size of the data-set is $n = NT$. With this notation, the model given in (1) to (4) has the following attributes: (i) $y$, $u$, $u_2$, and $\nu$ are $n \times 1$ vectors; (ii) $u_1$ and $\mu$ are $N \times 1$ vectors; (iii) $\tilde{L}$, $\tilde{A}$, and $\tilde{B}$ are $N \times N$ matrices of higher-order network effects (they could involve the dependent variable, the exogenous regressors, the time-invariant error component, and the time-variant residual). These $N \times N$ matrices will be defined below; (iv) $I_T$ is an identity matrix of size $T$, and $Z_\mu$ is a selector or assignment matrix which has the form $\iota_T \otimes I_N$, where $\iota_T$ is a column vector of ones of size $T$; (v) $X$ is an $n \times K$ matrix of exogenous regressors;[3] and (vi) $\beta$ is a vector of unknown parameters of dimension $K \times 1$.

Towards defining the matrices $\tilde{L}$, $\tilde{A}$, and $\tilde{B}$, it is useful to introduce the following notation; First, let $W_o^d$ be the $o$th-order (or -layer) weights matrix pertaining to the network effects which are relevant for $d = \{L, A, B\}$ and $o \in \mathfrak{O}_d$, where $\mathfrak{O}_d$ is a set of $O_d$ fixed and finite integer values. This notation allows the network matrices to be conceptually different and have potentially different order across $\tilde{L}$, $\tilde{A}$, and $\tilde{B}$. Second, For the unknown network parameters $\rho_o^d$ with $o \in \mathfrak{O}_d$. We can generally specify

$$\tilde{d} = \sum_{o \in \mathfrak{O}_d} \rho_o^d W_o^d.$$

For instance, this notation implies that for $d = L$, $\tilde{L} = \sum_{o \in \mathfrak{O}_L} \rho_o^L W_o^L$. Similarly for $d = A$ and $B$.

Note that $\mathfrak{O}_d$ governs the degree or the order of network effects with regard to concept $d$. $W_o^d$ parameterizes the (normalized) decay structure of network effects within order $o$, and $\rho_o^d$ scales this decay. It may be useful to think of a specific example with regard to $W_o^d$. Suppose that network effects emerge from some geographical or spatial neighborliness, and that the researcher is looking at data on firms which are all situated within 100 kilometers from each other. Then, considering rings of neighbors in 25-kilometer bands, $O^d = 4$, so that $o = 1, ..., 4$ would denote the four 25-kilometer rings. Moreover, $\rho_o^d W_o^d$ would measure the network effects among all pairs of firms within ring $o$. With the above notation, we generally allow the relevant network concepts to differ between $L$, $A$, and $B$. For instance, network effects could emerge through cultural similarity (e.g., different degrees of language similarity between countries or communities), through technological proximity (the similarity between technology parameters prevailing in different countries, regions,

[3]Note that $X$ is assumed to be doubly exogenous, i.e., with respect to both error components. This assumption is not as strong as it sounds, because X may contain averages of time-variant regressors (as in Mundlak (1978)), as well as higher-order network-weighted time-variant and time-invariant regressors.

or even firms), through endowment proximity (the similarity of factor endowments between different countries or regions), etc.

For $d = \{L, A, B\}$, let

$$d \;\; = \;\; I_N - \tilde{d}, \tag{5}$$

so that equations (3) and (4) can be written as

$$u_1 = A^{-1}\mu, \quad u_2 = (I_T \otimes B^{-1})\nu, \tag{6}$$

and the reduced form of the above model can be written as

$$y \;\; = \;\; (I_T \otimes L^{-1})(X\beta + u). \tag{7}$$

Assuming $\mu \sim IID(0, \sigma_\mu^2 I_N)$ and $\nu \sim IID(0, \sigma_\nu^2 I_n)$, where $\mu$ and $\nu$ are independent of each other and themselves. The variance-covariance matrix of $u$ with the above notation is

$$\begin{aligned}
\Omega_u &= E(uu') = Z_\mu E(u_1 u_1') Z_\mu' + E(u_2 u_2') \\
&= (\iota_T \otimes I_N)[\sigma_\mu^2 (A'A)^{-1}](\iota_T' \otimes I_N) + \sigma_\nu^2 [I_T \otimes (B'B)^{-1}] \\
&= \bar{J}_T \otimes \left(T\sigma_\mu^2 (A'A)^{-1} + \sigma_\nu^2 (B'B)^{-1}\right) + \sigma_\nu^2 \left(E_T \otimes (B'B)^{-1}\right),
\end{aligned}$$

The second equality uses the fact that $E(u_1 u_1') = \sigma_\mu^2 (A'A)^{-1}$ and $\hat{E}(u_2 u_2') = \sigma_\nu^2 [I_T \otimes (B'B)^{-1}]$. The third equality replaces the summing matrix over time $\iota_T \iota_T' = J_T$ by its idempotent counterpart $T\bar{J}_T$, with $\bar{J}_T = T^{-1}\iota_T \iota_T'$ denoting the averaging matrix over time. Also, $I_T$ is replaced by $E_T + \bar{J}_T$, where $E_T = I_T - \bar{J}_T$ is the deviations-from mean matrix, see Wansbeek and Kapteyn (1982) and Baltagi, Egger, and Pfaffermayr (2013).[4]

Under the normality assumption, the likelihood function of this model is given by

$$\begin{aligned}
L(\beta, \zeta) &= -\frac{n}{2} ln 2\pi - \frac{1}{2} ln |(I_T \otimes L^{-1})\Omega_u (I_T \otimes L^{-1})'| \\
&\quad -\frac{1}{2}(y - (I_T \otimes L^{-1})X\beta)'((I_T \otimes L^{-1})\Omega_u (I_T \otimes L^{-1})')^{-1}(y - (I_T \otimes L^{-1})X\beta) \\
&= -\frac{n}{2} ln 2\pi - \frac{1}{2} ln |T\sigma_\mu^2 (A'A)^{-1} + \sigma_\nu^2 (B'B)^{-1}| - \frac{T-1}{2} ln |\sigma_\nu^2 (B'B)^{-1}| + T ln |L| \\
&\quad -\frac{1}{2}((I_T \otimes L)y - X\beta)' \Omega_u^{-1} ((I_T \otimes L)y - X\beta),
\end{aligned}$$

[4] Note that the latter authors only considered first-order network processes and no network lag of the dependent variable. This model was generalized to include among other things serial correlation of the autoregressive moving average type by Lee and Yu (2011).

where $\zeta = (\sigma_\nu^2, \sigma_\mu^2, \rho_1^L, ..., \rho_O^L, \rho_1^A, ..., \rho_O^A, \rho_1^B, ..., \rho_O^B)$. There are a number of LR tests of interest for testing hypotheses regarding higher-order network effects. For instance, one could use LR tests to test against the absence of individual network effects, i.e., $\rho_o^d = 0$. Similarly, one could specify LR tests regarding joint hypotheses such as $\rho_1^d = ... = \rho_O^d = 0$ or $\rho_1^L = ... = \rho_O^L = \rho_1^A = ... = \rho_O^B = 0$. Moreover, such LR tests can be used to test the relevant order of the network effects. All of these tests could be done by using the difference in the maximized log-likelihood values between the restricted and unrestricted models. This LR test is asymptotically distributed as $\chi^2$ with degrees of freedom equal to the number of restrictions.

With this setting, we generally think of a situation, where the number of time periods $T$ is fixed (small), whereas the number of cross-sectional units $N$ grows asymptotically (is large). E.g., this is the case with most data-sets on firms or regions but also on individuals, where network effects are often studied.

## 2.2 Key assumptions

We make the following standard assumptions regarding the error components, the network weights matrices, and the parameters governing the network effects.

**Assumptions about the error components:** Apart from the IID and independence assumptions mentioned above regarding $\mu$ and $\nu$, we adopt the customary assumptions that the elements of $\mu$ and $\nu$ are bounded such that $E|\mu_i|^{4+\eta} < \infty$ and $E|\nu_{it}|^{4+\eta} < \infty$ for some $\eta > 0$ and for all $i$ and $t$ (see Badinger and Egger, 2015).

**Assumptions about the network weights matrices:** We assume that the network weights matrices $W_o^d$ for all $d = \{L, A, B\}$ and $o \in \mathfrak{O}_d$ are nonstochastic, have zero diagonal elements, and bounded off-diagonal elements which sum up to unity in each row or at most to unity for the minimum between the maximum row and column sums (see Kelejian and Prucha, 2010).

**Assumptions about the unknown network-effects parameters:** The matrices $d = \{L, A, B\}$ are invertible for all real parameter values $\rho_o^d$. We will make the sufficient assumption that $\sum_{o\in\mathfrak{O}_d} |\rho_o^d|$ is an element of a compact parameter space which is a subset of $[0, 1)$.

**Assumption about $X$:** The elements of $X$ are nonstochastic and uniformly bounded in $n$.

# 3 Monte Carlo simulations

## 3.1 Design

For the analysis of the small-sample performance of the proposed estimator, we consider the model given in equations (1)-(4), where $X = (\iota_n, x)$ with $x = \zeta_i + z_{it}$, $\zeta_i \sim IIDU[-7.5, 7.5]$, and $z_{it} \sim IIDU[-5, 5]$ as in Baltagi, Egger, and Pfaffermayr (2013), where $U[\cdot]$ denotes the uniform distribution of a given compact interval. The two processes of $\zeta_i$ and $z_{it}$ are assumed to be independent and they are kept fixed across the Monte Carlo draws. Regarding the parameters on $X$, we assume $\beta = (5, 0.5)'$, where the first parameter denotes the constant.

Moreover, in the Monte Carlo simulations we consider the case where the order of the network relations is two, $O_d = 2$, for every concept $d \in \{L, A, B\}$. Moreover, we assume that the weights matrices are identical between the $d$ so that $W_o^d = W_o$ for all $d$. For the sake of simplicity, $W_1$ and $W_2$ will take a very simple form on a so-called wrap-around lattice: $W_1$ has a normalized three-before-and-three-behind design, and $W_2$ has a normalized four-to-six-before and four-to-six-behind design. By this choice, the positive entries of $W_1$ and $W_2$ are 1/6 and non-overlapping. For the purpose of illustration, we display these matrices in a condensed way here.

$$W_1 = \begin{pmatrix} 0 & 1/6 & 1/6 & 1/6 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 1/6 & 1/6 & 1/6 \\ 1/6 & 0 & 1/6 & 1/6 & 1/6 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 1/6 & 1/6 \\ 1/6 & 1/6 & 0 & 1/6 & 1/6 & 1/6 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 1/6 \\ 1/6 & 1/6 & 1/6 & 0 & 1/6 & 1/6 & 1/6 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1/6 & 1/6 & 1/6 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 1/6 & 1/6 & 1/6 & 0 \\ \end{pmatrix},$$

$$
W_2 = \begin{pmatrix}
0 & 0 & 0 & 0 & 1/6 & 1/6 & 1/6 & \cdots & 1/6 & 1/6 & 1/6 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1/6 & 1/6 & 1/6 & \cdots & 1/6 & 1/6 & 1/6 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1/6 & 1/6 & 1/6 & \cdots & 1/6 & 1/6 & 1/6 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/6 & 1/6 & 1/6 & \cdots & 1/6 & 1/6 & 1/6 \\
1/6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/6 & 1/6 & 1/6 & \cdots & 1/6 & 1/6 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 1/6 & 1/6 & 1/6 & \cdots & 1/6 & 1/6 & 1/6 & 0 & 0 & 0 & 0
\end{pmatrix}, \quad (8)
$$

The wrap-around, banded nature of the two respective lattices underlying $W_1$ and $W_2$ is apparent from the last column elements in the first three rows and the first three column elements of the last three rows of $W_1$. Similarly, it is apparent from the structure of rows four to six from the top and from the bottom of $W_2$. All row sums of both $W_1$ and $W_2$ sum up to unity.

For the parameters governing the strength of network relations, we postulate three configurations for each concept $d = \{L, A, B\}$ each of $\rho^d = \{(0.6, 0.2); (0.6, 0.0); (0.0, 0.0)\}$ and again use all permutations. Hence, with three parameter configurations of $\rho^d$ and three concepts in $d = \{L, A, B\}$ there are $3^3 = 27$ permutations of configurations.

Regarding $T$ and $N$, we use all permutations of the number of time periods of $T = \{5; 10\}$ and the number of cross-sectional units of $N = \{100; 200\}$.

With respect to the disturbances, we follow Baltagi, Egger, and Pfaffermayr (2013) in assuming that the individual-specific effects are drawn from a normal distribution so that $\mu_i \sim IIDN(0, 20\theta)$, and the remainder error is distributed as $\nu_{it} \sim IIDN(0, 20(1-\theta))$, where $0 \leq \theta = \frac{\sigma_\mu^2}{\sigma_\mu^2 + \sigma_\nu^2} \leq 1$ parameterizes the proportion of heterogeneity of the individual-specific effects in the total variance of the disturbances. We set $\theta = \{0.25; 0.5\}$.

This means that there are all together 216 configurations of the design or experiments. For each experiment, we perform $1,000$ replications and report the bias and root mean squared error of the results.

## 3.2 Results

Tables 1-8 report the bias of the parameter estimates across the $1,000$ replications, while Tables 9-16 report the corresponding root mean-squared error (RMSE) across the $1,000$ replications. For each table, the heading gives the corresponding $N, T$ and $\theta$. The block labelled *Configuration* reports the true values of $\sigma_\mu^2$ and $\sigma_\nu^2$ as well as $\rho_o^d$ for $d \in \{L, A, B\}$ and $o \in \{1, 2\}$. We do not provide the true values of the parameters in $\beta$, as they are fixed at $(5, 0.5)$ for all experiments.

A comparison of Table 1 ($N = 100$, $T = 5$) with Table 2 ($N = 100$, $T = 10$), Table 3 ($N = 200$, $T = 5$), and Table 4 ($N = 200$, $T = 10$) suggests the following conclusions for a configuration where $\theta = 0.25$. First, the bias of the parameters is already quite low in Table 1, and it diminishes as $T$ and, in particular, as $N$ increases. The companion comparison regarding the RMSE in Tables 9-12 suggests that also the RMSE declines as $T$ and, in particular, as $N$ increases.

With $\theta = 0.5$ instead of $\theta = 0.25$, i.e., doubling the heterogeneity across the individual effects, the bias in $\rho_o^A$ and $\rho_o^B$ tends to be slightly larger in absolute value. To see this, compare the results in Tables 5-8 (where $\theta = 0.5$) with the corresponding ones in Tables 1-4 (where $\theta = 0.25$). However, this bias improves as the sample size increases. Compare the results in Tables 5 and 1, on the one hand, and those in Tables 8 and 4, respectively. A comparison of the results in Tables 9-12 with the ones in Tables 13-16 regarding the RMSE suggests the following conclusions. First, the RMSE tends to be somewhat smaller for $\rho_o^d$ for $d \in \{L, A, B\}$ and $o \in \{1, 2\}$, where $\theta$ is higher. Second, the RMSE tends to decline as $T$, and particularly, as $N$ increases, irrespective of the level of $\theta$.

Overall, we conclude that the small-sample performance of the estimator appears to be quite good and we recommend it for use in applications with even modest sample sizes.

## 3.3 Likelihood-ratio tests

In the previous subsection, we documented the performance of the proposed estimator with regard to point estimates with an emphasis on bias and RMSE. In this subsection, we consider selected testing results, using likelihood-ratio (LR) tests and four alternative null hypotheses: $H_0^1 : \rho_2^L = 0$, a second one is $H_0^2 : \rho_2^A = 0$, a third one is that $H_0^3 : \rho_2^B = 0$, and the fourth one is $H_0^4 : \rho_2^L = \rho_2^A = \rho_2^B = 0$. These test that there is no second order network effect in the lagged dependent variable only, the random individual time-invariant effect only, the remainder disturbance term only, or all of them jointly.

Table 1: Bias of Parameters, $N = 100$, $T = 5$, $\theta = 0.25$

| **Configuration** | | | | | | | | **Bias** | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\rho_1^L$ | $\rho_2^L$ | $\rho_1^A$ | $\rho_2^A$ | $\rho_1^B$ | $\rho_2^B$ | $\sigma_\mu^2$ | $\sigma_\nu^2$ | $\beta_0$ | $\beta_1$ | $\sigma_\mu^2$ | $\sigma_\nu^2$ | $\rho_1^L$ | $\rho_2^L$ | $\rho_1^A$ | $\rho_2^A$ | $\rho_1^B$ | $\rho_2^B$ |
| 0.6 | 0.2 | 0.6 | 0.2 | 0.6 | 0.2 | 5 | 15 | -0.294 | -0.002 | -0.437 | -0.475 | 0.010 | -0.015 | -0.173 | -0.072 | -0.057 | -0.001 |
| 0.6 | 0.2 | 0.6 | 0.2 | 0.6 | 0 | 5 | 15 | -0.190 | -0.003 | -0.335 | -0.521 | 0.016 | -0.020 | -0.127 | -0.041 | -0.071 | 0.004 |
| 0.6 | 0.2 | 0.6 | 0.2 | 0 | 0 | 5 | 15 | 0.038 | -0.002 | -0.248 | -0.531 | -0.012 | 0.008 | -0.043 | -0.048 | -0.034 | -0.025 |
| 0.6 | 0.2 | 0.6 | 0 | 0.6 | 0.2 | 5 | 15 | -0.065 | -0.002 | -0.480 | -0.477 | 0.014 | -0.020 | -0.228 | -0.057 | -0.058 | 0.004 |
| 0.6 | 0.2 | 0.6 | 0 | 0.6 | 0 | 5 | 15 | 0.107 | -0.002 | -0.445 | -0.497 | 0.021 | -0.028 | -0.211 | -0.035 | -0.070 | 0.013 |
| 0.6 | 0.2 | 0.6 | 0 | 0 | 0 | 5 | 15 | 0.168 | -0.001 | -0.263 | -0.514 | -0.005 | -0.002 | -0.090 | -0.026 | -0.037 | -0.012 |
| 0.6 | 0.2 | 0 | 0 | 0.6 | 0.2 | 5 | 15 | -0.186 | -0.003 | -0.505 | -0.482 | 0.001 | 0.000 | -0.158 | -0.050 | -0.046 | -0.015 |
| 0.6 | 0.2 | 0 | 0 | 0.6 | 0 | 5 | 15 | 0.275 | -0.004 | -0.601 | -0.477 | 0.008 | -0.021 | -0.211 | -0.072 | -0.056 | 0.012 |
| 0.6 | 0.2 | 0 | 0 | 0 | 0 | 5 | 15 | 0.289 | -0.001 | -0.525 | -0.441 | -0.014 | 0.002 | -0.146 | -0.085 | -0.020 | -0.012 |
| 0.6 | 0 | 0.6 | 0.2 | 0.6 | 0.2 | 5 | 15 | -0.490 | -0.002 | -0.439 | -0.500 | -0.001 | 0.022 | -0.175 | -0.113 | -0.050 | -0.035 |
| 0.6 | 0 | 0.6 | 0.2 | 0.6 | 0 | 5 | 15 | -0.324 | -0.003 | -0.373 | -0.575 | -0.001 | 0.008 | -0.124 | -0.077 | -0.067 | -0.025 |
| 0.6 | 0 | 0.6 | 0.2 | 0 | 0 | 5 | 15 | -0.004 | -0.003 | -0.228 | -0.569 | -0.022 | 0.011 | -0.042 | -0.057 | -0.033 | -0.032 |
| 0.6 | 0 | 0.6 | 0 | 0.6 | 0.2 | 5 | 15 | -0.241 | -0.002 | -0.507 | -0.497 | -0.001 | 0.010 | -0.229 | -0.092 | -0.049 | -0.025 |
| 0.6 | 0 | 0.6 | 0 | 0.6 | 0 | 5 | 15 | 0.066 | -0.003 | -0.473 | -0.547 | -0.004 | -0.009 | -0.193 | -0.058 | -0.058 | -0.008 |
| 0.6 | 0 | 0.6 | 0 | 0 | 0 | 5 | 15 | 0.188 | -0.003 | -0.269 | -0.554 | -0.019 | 0.000 | -0.084 | -0.032 | -0.031 | -0.018 |
| 0.6 | 0 | 0 | 0 | 0.6 | 0.2 | 5 | 15 | -0.062 | -0.002 | -0.511 | -0.488 | -0.020 | 0.018 | -0.143 | -0.071 | -0.032 | -0.034 |
| 0.6 | 0 | 0 | 0 | 0.6 | 0 | 5 | 15 | 0.243 | -0.003 | -0.586 | -0.519 | -0.020 | -0.003 | -0.191 | -0.091 | -0.041 | -0.013 |
| 0.6 | 0 | 0 | 0 | 0 | 0 | 5 | 15 | 0.293 | -0.002 | -0.497 | -0.494 | -0.026 | 0.002 | -0.145 | -0.083 | -0.014 | -0.014 |
| 0 | 0 | 0.6 | 0.2 | 0.6 | 0.2 | 5 | 15 | -0.271 | -0.002 | -0.438 | -0.462 | 0.014 | 0.015 | -0.177 | -0.111 | -0.048 | -0.023 |
| 0 | 0 | 0.6 | 0.2 | 0.6 | 0 | 5 | 15 | -0.239 | -0.003 | -0.351 | -0.561 | 0.013 | 0.008 | -0.127 | -0.082 | -0.066 | -0.025 |
| 0 | 0 | 0.6 | 0.2 | 0 | 0 | 5 | 15 | -0.026 | -0.006 | -0.285 | -0.726 | -0.023 | 0.005 | -0.044 | -0.059 | -0.039 | -0.034 |
| 0 | 0 | 0.6 | 0 | 0.6 | 0.2 | 5 | 15 | -0.130 | -0.002 | -0.494 | -0.461 | 0.010 | 0.006 | -0.224 | -0.093 | -0.045 | -0.017 |
| 0 | 0 | 0.6 | 0 | 0.6 | 0 | 5 | 15 | 0.008 | -0.004 | -0.466 | -0.577 | 0.000 | -0.008 | -0.196 | -0.077 | -0.057 | -0.015 |
| 0 | 0 | 0.6 | 0 | 0 | 0 | 5 | 15 | 0.176 | -0.007 | -0.324 | -0.723 | -0.029 | -0.013 | -0.084 | -0.040 | -0.032 | -0.018 |
| 0 | 0 | 0 | 0 | 0.6 | 0.2 | 5 | 15 | 0.081 | -0.004 | -0.505 | -0.540 | -0.020 | 0.000 | -0.131 | -0.063 | -0.029 | -0.021 |
| 0 | 0 | 0 | 0 | 0.6 | 0 | 5 | 15 | 0.203 | -0.006 | -0.605 | -0.635 | -0.029 | -0.012 | -0.180 | -0.094 | -0.038 | -0.016 |
| 0 | 0 | 0 | 0 | 0 | 0 | 5 | 15 | 0.334 | -0.007 | -0.569 | -0.690 | -0.048 | -0.018 | -0.133 | -0.087 | -0.009 | -0.010 |

Table 2: Bias of Parameters, $N = 100$, $T = 10$, $\theta = 0.25$

| **Configuration** | | | | | | | | **Bias** | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\rho_1^L$ | $\rho_2^L$ | $\rho_1^A$ | $\rho_2^A$ | $\rho_1^B$ | $\rho_2^B$ | $\sigma_\mu^2$ | $\sigma_\nu^2$ | $\beta_0$ | $\beta_1$ | $\sigma_\mu^2$ | $\sigma_\nu^2$ | $\rho_1^L$ | $\rho_2^L$ | $\rho_1^A$ | $\rho_2^A$ | $\rho_1^B$ | $\rho_2^B$ |
| 0.6 | 0.2 | 0.6 | 0.2 | 0.6 | 0.2 | 5 | 15 | -0.115 | -0.004 | -0.322 | -0.289 | 0.000 | -0.006 | -0.112 | -0.057 | -0.029 | -0.006 |
| 0.6 | 0.2 | 0.6 | 0.2 | 0.6 | 0 | 5 | 15 | -0.076 | -0.003 | -0.254 | -0.278 | 0.006 | -0.011 | -0.079 | -0.034 | -0.037 | 0.002 |
| 0.6 | 0.2 | 0.6 | 0.2 | 0 | 0 | 5 | 15 | 0.095 | -0.003 | -0.208 | -0.247 | -0.007 | 0.000 | -0.041 | -0.037 | -0.011 | -0.005 |
| 0.6 | 0.2 | 0.6 | 0 | 0.6 | 0.2 | 5 | 15 | 0.262 | -0.004 | -0.385 | -0.275 | -0.003 | -0.013 | -0.150 | -0.049 | -0.024 | 0.001 |
| 0.6 | 0.2 | 0.6 | 0 | 0.6 | 0 | 5 | 15 | 0.192 | -0.003 | -0.319 | -0.270 | 0.008 | -0.019 | -0.129 | -0.033 | -0.036 | 0.011 |
| 0.6 | 0.2 | 0.6 | 0 | 0 | 0 | 5 | 15 | 0.215 | -0.003 | -0.235 | -0.228 | -0.006 | -0.003 | -0.068 | -0.026 | -0.009 | 0.000 |
| 0.6 | 0.2 | 0 | 0 | 0.6 | 0.2 | 5 | 15 | 0.253 | -0.004 | -0.483 | -0.255 | -0.005 | -0.008 | -0.161 | -0.052 | -0.020 | -0.004 |
| 0.6 | 0.2 | 0 | 0 | 0.6 | 0 | 5 | 15 | 0.362 | -0.004 | -0.531 | -0.252 | 0.003 | -0.018 | -0.196 | -0.070 | -0.027 | 0.014 |
| 0.6 | 0.2 | 0 | 0 | 0 | 0 | 5 | 15 | 0.279 | -0.003 | -0.408 | -0.203 | -0.007 | -0.005 | -0.125 | -0.063 | -0.006 | 0.003 |
| 0.6 | 0 | 0.6 | 0.2 | 0.6 | 0.2 | 5 | 15 | -0.272 | -0.004 | -0.311 | -0.306 | -0.002 | 0.015 | -0.115 | -0.075 | -0.029 | -0.024 |
| 0.6 | 0 | 0.6 | 0.2 | 0.6 | 0 | 5 | 15 | -0.122 | -0.004 | -0.270 | -0.357 | -0.005 | 0.004 | -0.085 | -0.048 | -0.037 | -0.015 |
| 0.6 | 0 | 0.6 | 0.2 | 0 | 0 | 5 | 15 | 0.056 | -0.004 | -0.206 | -0.278 | -0.016 | 0.004 | -0.039 | -0.043 | -0.009 | -0.013 |
| 0.6 | 0 | 0.6 | 0 | 0.6 | 0.2 | 5 | 15 | -0.078 | -0.004 | -0.393 | -0.301 | -0.006 | 0.009 | -0.157 | -0.073 | -0.025 | -0.019 |
| 0.6 | 0 | 0.6 | 0 | 0.6 | 0 | 5 | 15 | 0.168 | -0.005 | -0.367 | -0.354 | -0.009 | -0.008 | -0.132 | -0.048 | -0.032 | -0.003 |
| 0.6 | 0 | 0.6 | 0 | 0 | 0 | 5 | 15 | 0.204 | -0.004 | -0.237 | -0.271 | -0.015 | -0.003 | -0.067 | -0.029 | -0.008 | -0.003 |
| 0.6 | 0 | 0 | 0 | 0.6 | 0.2 | 5 | 15 | 0.038 | -0.004 | -0.483 | -0.278 | -0.012 | 0.008 | -0.154 | -0.066 | -0.017 | -0.018 |
| 0.6 | 0 | 0 | 0 | 0.6 | 0 | 5 | 15 | 0.414 | -0.005 | -0.552 | -0.324 | -0.019 | -0.014 | -0.185 | -0.072 | -0.018 | 0.004 |
| 0.6 | 0 | 0 | 0 | 0 | 0 | 5 | 15 | 0.278 | -0.003 | -0.422 | -0.246 | -0.015 | -0.007 | -0.126 | -0.064 | -0.002 | 0.004 |
| 0 | 0 | 0.6 | 0.2 | 0.6 | 0.2 | 5 | 15 | -0.243 | -0.002 | -0.289 | -0.208 | 0.026 | 0.014 | -0.119 | -0.069 | -0.037 | -0.011 |
| 0 | 0 | 0.6 | 0.2 | 0.6 | 0 | 5 | 15 | -0.225 | -0.003 | -0.251 | -0.273 | 0.024 | 0.013 | -0.092 | -0.056 | -0.048 | -0.019 |
| 0 | 0 | 0.6 | 0.2 | 0 | 0 | 5 | 15 | 0.023 | -0.006 | -0.243 | -0.386 | -0.012 | 0.000 | -0.044 | -0.043 | -0.019 | -0.014 |
| 0 | 0 | 0.6 | 0 | 0.6 | 0.2 | 5 | 15 | -0.154 | -0.002 | -0.373 | -0.221 | 0.020 | 0.008 | -0.164 | -0.072 | -0.033 | -0.008 |
| 0 | 0 | 0.6 | 0 | 0.6 | 0 | 5 | 15 | -0.109 | -0.003 | -0.350 | -0.293 | 0.014 | 0.006 | -0.142 | -0.063 | -0.041 | -0.015 |
| 0 | 0 | 0.6 | 0 | 0 | 0 | 5 | 15 | 0.124 | -0.006 | -0.293 | -0.397 | -0.017 | -0.009 | -0.070 | -0.034 | -0.015 | -0.005 |
| 0 | 0 | 0 | 0 | 0.6 | 0.2 | 5 | 15 | -0.045 | -0.003 | -0.472 | -0.247 | 0.007 | 0.002 | -0.156 | -0.062 | -0.024 | -0.006 |
| 0 | 0 | 0 | 0 | 0.6 | 0 | 5 | 15 | 0.022 | -0.004 | -0.552 | -0.309 | -0.002 | -0.002 | -0.192 | -0.088 | -0.028 | -0.010 |
| 0 | 0 | 0 | 0 | 0 | 0 | 5 | 15 | 0.175 | -0.006 | -0.465 | -0.372 | -0.019 | -0.015 | -0.127 | -0.064 | -0.009 | 0.002 |

Table 3: Bias of Parameters, $N = 200$, $T = 5$, $\theta = 0.25$

| **Configuration** | | | | | | | | **Bias** | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\rho_1^L$ | $\rho_2^L$ | $\rho_1^A$ | $\rho_2^A$ | $\rho_1^B$ | $\rho_2^B$ | $\sigma_\mu^2$ | $\sigma_\nu^2$ | $\beta_0$ | $\beta_1$ | $\sigma_\mu^2$ | $\sigma_\nu^2$ | $\rho_1^L$ | $\rho_2^L$ | $\rho_1^A$ | $\rho_2^A$ | $\rho_1^B$ | $\rho_2^B$ |
| 0.6 | 0.2 | 0.6 | 0.2 | 0.6 | 0.2 | 5 | 15 | -0.191 | -0.004 | -0.264 | -0.240 | 0.002 | -0.001 | -0.081 | -0.039 | -0.025 | -0.014 |
| 0.6 | 0.2 | 0.6 | 0.2 | 0.6 | 0 | 5 | 15 | -0.002 | -0.003 | -0.199 | -0.234 | 0.001 | -0.007 | -0.049 | -0.015 | -0.027 | -0.004 |
| 0.6 | 0.2 | 0.6 | 0.2 | 0 | 0 | 5 | 15 | 0.138 | -0.002 | -0.149 | -0.201 | -0.009 | 0.001 | -0.016 | -0.018 | -0.007 | -0.008 |
| 0.6 | 0.2 | 0.6 | 0 | 0.6 | 0.2 | 5 | 15 | 0.064 | -0.004 | -0.309 | -0.230 | 0.002 | -0.008 | -0.122 | -0.038 | -0.024 | -0.006 |
| 0.6 | 0.2 | 0.6 | 0 | 0.6 | 0 | 5 | 15 | 0.187 | -0.003 | -0.264 | -0.210 | 0.005 | -0.015 | -0.096 | -0.013 | -0.028 | 0.004 |
| 0.6 | 0.2 | 0.6 | 0 | 0 | 0 | 5 | 15 | 0.211 | -0.002 | -0.159 | -0.186 | -0.006 | -0.002 | -0.035 | -0.009 | -0.007 | -0.004 |
| 0.6 | 0.2 | 0 | 0 | 0.6 | 0.2 | 5 | 15 | 0.125 | -0.004 | -0.392 | -0.226 | -0.004 | -0.004 | -0.125 | -0.041 | -0.018 | -0.012 |
| 0.6 | 0.2 | 0 | 0 | 0.6 | 0 | 5 | 15 | 0.401 | -0.003 | -0.438 | -0.217 | -0.004 | -0.013 | -0.150 | -0.053 | -0.015 | 0.005 |
| 0.6 | 0.2 | 0 | 0 | 0 | 0 | 5 | 15 | 0.278 | -0.002 | -0.317 | -0.156 | -0.005 | -0.006 | -0.090 | -0.039 | -0.002 | 0.004 |
| 0.6 | 0 | 0.6 | 0.2 | 0.6 | 0.2 | 5 | 15 | -0.288 | -0.003 | -0.261 | -0.245 | 0.000 | 0.019 | -0.089 | -0.062 | -0.027 | -0.032 |
| 0.6 | 0 | 0.6 | 0.2 | 0.6 | 0 | 5 | 15 | -0.103 | -0.004 | -0.214 | -0.307 | -0.006 | 0.008 | -0.051 | -0.033 | -0.031 | -0.023 |
| 0.6 | 0 | 0.6 | 0.2 | 0 | 0 | 5 | 15 | 0.112 | -0.003 | -0.154 | -0.230 | -0.017 | 0.004 | -0.013 | -0.026 | -0.006 | -0.016 |
| 0.6 | 0 | 0.6 | 0 | 0.6 | 0.2 | 5 | 15 | -0.146 | -0.003 | -0.318 | -0.244 | -0.004 | 0.013 | -0.130 | -0.064 | -0.023 | -0.027 |
| 0.6 | 0 | 0.6 | 0 | 0.6 | 0 | 5 | 15 | 0.120 | -0.004 | -0.278 | -0.287 | -0.009 | -0.004 | -0.096 | -0.030 | -0.026 | -0.010 |
| 0.6 | 0 | 0.6 | 0 | 0 | 0 | 5 | 15 | 0.223 | -0.003 | -0.175 | -0.220 | -0.016 | -0.003 | -0.032 | -0.014 | -0.004 | -0.006 |
| 0.6 | 0 | 0 | 0 | 0.6 | 0.2 | 5 | 15 | -0.062 | -0.003 | -0.392 | -0.229 | -0.009 | 0.012 | -0.127 | -0.059 | -0.018 | -0.028 |
| 0.6 | 0 | 0 | 0 | 0.6 | 0 | 5 | 15 | 0.264 | -0.004 | -0.468 | -0.261 | -0.015 | -0.007 | -0.157 | -0.067 | -0.017 | -0.007 |
| 0.6 | 0 | 0 | 0 | 0 | 0 | 5 | 15 | 0.231 | -0.002 | -0.329 | -0.185 | -0.015 | -0.004 | -0.086 | -0.045 | -0.001 | -0.003 |
| 0 | 0 | 0.6 | 0.2 | 0.6 | 0.2 | 5 | 15 | -0.170 | -0.002 | -0.238 | -0.170 | 0.015 | 0.015 | -0.086 | -0.050 | -0.025 | -0.016 |
| 0 | 0 | 0.6 | 0.2 | 0.6 | 0 | 5 | 15 | -0.150 | -0.003 | -0.198 | -0.247 | 0.012 | 0.015 | -0.054 | -0.035 | -0.037 | -0.024 |
| 0 | 0 | 0.6 | 0.2 | 0 | 0 | 5 | 15 | 0.041 | -0.005 | -0.187 | -0.345 | -0.016 | 0.004 | -0.018 | -0.026 | -0.016 | -0.021 |
| 0 | 0 | 0.6 | 0 | 0.6 | 0.2 | 5 | 15 | -0.156 | -0.002 | -0.309 | -0.172 | 0.015 | 0.015 | -0.133 | -0.064 | -0.025 | -0.017 |
| 0 | 0 | 0.6 | 0 | 0.6 | 0 | 5 | 15 | -0.069 | -0.003 | -0.283 | -0.260 | 0.005 | 0.008 | -0.103 | -0.043 | -0.031 | -0.020 |
| 0 | 0 | 0.6 | 0 | 0 | 0 | 5 | 15 | 0.126 | -0.005 | -0.211 | -0.351 | -0.021 | -0.006 | -0.035 | -0.018 | -0.010 | -0.012 |
| 0 | 0 | 0 | 0 | 0.6 | 0.2 | 5 | 15 | -0.075 | -0.003 | -0.390 | -0.193 | 0.006 | 0.009 | -0.127 | -0.051 | -0.021 | -0.015 |
| 0 | 0 | 0 | 0 | 0.6 | 0 | 5 | 15 | -0.029 | -0.004 | -0.464 | -0.258 | -0.001 | 0.006 | -0.160 | -0.074 | -0.026 | -0.019 |
| 0 | 0 | 0 | 0 | 0 | 0 | 5 | 15 | 0.133 | -0.005 | -0.368 | -0.314 | -0.019 | -0.008 | -0.090 | -0.045 | -0.008 | -0.007 |

Table 4: Bias of Parameters, $N = 200$, $T = 10$, $\theta = 0.25$

| **Configuration** | | | | | | | | **Bias** | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\rho_1^L$ | $\rho_2^L$ | $\rho_1^A$ | $\rho_2^A$ | $\rho_1^B$ | $\rho_2^B$ | $\sigma_\mu^2$ | $\sigma_\nu^2$ | $\beta_0$ | $\beta_1$ | $\sigma_\mu^2$ | $\sigma_\nu^2$ | $\rho_1^L$ | $\rho_2^L$ | $\rho_1^A$ | $\rho_2^A$ | $\rho_1^B$ | $\rho_2^B$ |
| 0.6 | 0.2 | 0.6 | 0.2 | 0.6 | 0.2 | 5 | 15 | 0.106 | -0.001 | -0.167 | -0.131 | -0.010 | 0.001 | -0.037 | -0.029 | -0.006 | -0.011 |
| 0.6 | 0.2 | 0.6 | 0.2 | 0.6 | 0 | 5 | 15 | 0.210 | 0.000 | -0.151 | -0.120 | -0.008 | -0.005 | -0.027 | -0.016 | -0.007 | 0.001 |
| 0.6 | 0.2 | 0.6 | 0.2 | 0 | 0 | 5 | 15 | 0.156 | 0.000 | -0.118 | -0.061 | -0.010 | 0.002 | -0.013 | -0.017 | 0.006 | -0.002 |
| 0.6 | 0.2 | 0.6 | 0 | 0.6 | 0.2 | 5 | 15 | 0.438 | -0.001 | -0.208 | -0.133 | -0.014 | -0.007 | -0.061 | -0.023 | -0.002 | -0.002 |
| 0.6 | 0.2 | 0.6 | 0 | 0.6 | 0 | 5 | 15 | 0.293 | 0.000 | -0.181 | -0.114 | -0.006 | -0.008 | -0.052 | -0.012 | -0.008 | 0.004 |
| 0.6 | 0.2 | 0.6 | 0 | 0 | 0 | 5 | 15 | 0.209 | 0.000 | -0.131 | -0.056 | -0.009 | 0.000 | -0.025 | -0.011 | 0.006 | 0.002 |
| 0.6 | 0.2 | 0 | 0 | 0.6 | 0.2 | 5 | 15 | 0.446 | -0.001 | -0.330 | -0.126 | -0.017 | -0.004 | -0.101 | -0.042 | 0.001 | -0.006 |
| 0.6 | 0.2 | 0 | 0 | 0.6 | 0 | 5 | 15 | 0.524 | -0.001 | -0.336 | -0.100 | -0.010 | -0.012 | -0.118 | -0.047 | -0.001 | 0.011 |
| 0.6 | 0.2 | 0 | 0 | 0 | 0 | 5 | 15 | 0.230 | 0.000 | -0.219 | -0.044 | -0.009 | -0.001 | -0.056 | -0.029 | 0.008 | 0.003 |
| 0.6 | 0 | 0.6 | 0.2 | 0.6 | 0.2 | 5 | 15 | -0.191 | 0.000 | -0.172 | -0.129 | -0.009 | 0.019 | -0.044 | -0.045 | -0.009 | -0.027 |
| 0.6 | 0 | 0.6 | 0.2 | 0.6 | 0 | 5 | 15 | 0.067 | -0.001 | -0.167 | -0.181 | -0.017 | 0.006 | -0.027 | -0.027 | -0.009 | -0.015 |
| 0.6 | 0 | 0.6 | 0.2 | 0 | 0 | 5 | 15 | 0.141 | 0.000 | -0.124 | -0.079 | -0.016 | 0.001 | -0.011 | -0.020 | 0.006 | -0.004 |
| 0.6 | 0 | 0.6 | 0 | 0.6 | 0.2 | 5 | 15 | -0.033 | -0.001 | -0.218 | -0.135 | -0.014 | 0.014 | -0.068 | -0.046 | -0.004 | -0.022 |
| 0.6 | 0 | 0.6 | 0 | 0.6 | 0 | 5 | 15 | 0.292 | -0.002 | -0.202 | -0.181 | -0.022 | -0.004 | -0.047 | -0.020 | -0.003 | -0.006 |
| 0.6 | 0 | 0.6 | 0 | 0 | 0 | 5 | 15 | 0.215 | 0.000 | -0.138 | -0.075 | -0.015 | -0.004 | -0.024 | -0.012 | 0.008 | 0.002 |
| 0.6 | 0 | 0 | 0 | 0.6 | 0.2 | 5 | 15 | -0.001 | -0.001 | -0.331 | -0.121 | -0.016 | 0.014 | -0.103 | -0.060 | 0.000 | -0.023 |
| 0.6 | 0 | 0 | 0 | 0.6 | 0 | 5 | 15 | 0.393 | -0.002 | -0.368 | -0.158 | -0.026 | -0.007 | -0.114 | -0.054 | 0.004 | -0.002 |
| 0.6 | 0 | 0 | 0 | 0 | 0 | 5 | 15 | 0.232 | 0.000 | -0.230 | -0.062 | -0.015 | -0.005 | -0.055 | -0.027 | 0.010 | 0.004 |
| 0 | 0 | 0.6 | 0.2 | 0.6 | 0.2 | 5 | 15 | -0.092 | 0.000 | -0.159 | -0.099 | 0.001 | 0.009 | -0.040 | -0.029 | -0.008 | -0.009 |
| 0 | 0 | 0.6 | 0.2 | 0.6 | 0 | 5 | 15 | -0.089 | -0.001 | -0.150 | -0.140 | 0.001 | 0.011 | -0.033 | -0.026 | -0.016 | -0.016 |
| 0 | 0 | 0.6 | 0.2 | 0 | 0 | 5 | 15 | 0.050 | -0.002 | -0.150 | -0.173 | -0.017 | 0.004 | -0.013 | -0.023 | 0.001 | -0.012 |
| 0 | 0 | 0.6 | 0 | 0.6 | 0.2 | 5 | 15 | -0.065 | 0.000 | -0.208 | -0.101 | 0.000 | 0.007 | -0.072 | -0.037 | -0.008 | -0.008 |
| 0 | 0 | 0.6 | 0 | 0.6 | 0 | 5 | 15 | -0.041 | -0.001 | -0.194 | -0.148 | -0.002 | 0.006 | -0.058 | -0.030 | -0.014 | -0.013 |
| 0 | 0 | 0.6 | 0 | 0 | 0 | 5 | 15 | 0.098 | -0.002 | -0.170 | -0.177 | -0.019 | -0.002 | -0.025 | -0.017 | 0.003 | -0.007 |
| 0 | 0 | 0 | 0 | 0.6 | 0.2 | 5 | 15 | -0.026 | -0.001 | -0.334 | -0.108 | -0.005 | 0.005 | -0.110 | -0.052 | -0.005 | -0.008 |
| 0 | 0 | 0 | 0 | 0.6 | 0 | 5 | 15 | 0.016 | -0.001 | -0.372 | -0.149 | -0.009 | 0.003 | -0.126 | -0.066 | -0.008 | -0.011 |
| 0 | 0 | 0 | 0 | 0 | 0 | 5 | 15 | 0.106 | -0.002 | -0.266 | -0.162 | -0.018 | -0.004 | -0.059 | -0.034 | 0.004 | -0.003 |

Table 5: Bias of Parameters, $N = 100$, $T = 5$, $\theta = 0.5$

| **Configuration** | | | | | | | | **Bias** | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\rho_1^L$ | $\rho_2^L$ | $\rho_1^A$ | $\rho_2^A$ | $\rho_1^B$ | $\rho_2^B$ | $\sigma_\mu^2$ | $\sigma_\nu^2$ | $\beta_0$ | $\beta_1$ | $\sigma_\mu^2$ | $\sigma_\nu^2$ | $\rho_1^L$ | $\rho_2^L$ | $\rho_1^A$ | $\rho_2^A$ | $\rho_1^B$ | $\rho_2^B$ |
| 0.6 | 0.2 | 0.6 | 0.2 | 0.6 | 0.2 | 10 | 10 | -0.419 | -0.003 | -0.543 | -0.329 | 0.007 | -0.013 | -0.104 | -0.042 | -0.054 | -0.003 |
| 0.6 | 0.2 | 0.6 | 0.2 | 0.6 | 0 | 10 | 10 | -0.062 | -0.003 | -0.481 | -0.344 | 0.005 | -0.019 | -0.082 | -0.025 | -0.060 | 0.005 |
| 0.6 | 0.2 | 0.6 | 0.2 | 0 | 0 | 10 | 10 | -0.012 | -0.002 | -0.386 | -0.348 | -0.015 | 0.006 | -0.040 | -0.040 | -0.027 | -0.021 |
| 0.6 | 0.2 | 0.6 | 0 | 0.6 | 0.2 | 10 | 10 | 0.248 | -0.002 | -0.641 | -0.325 | 0.005 | -0.024 | -0.143 | -0.030 | -0.049 | 0.007 |
| 0.6 | 0.2 | 0.6 | 0 | 0.6 | 0 | 10 | 10 | 0.897 | -0.003 | -0.581 | -0.344 | -0.003 | -0.036 | -0.118 | -0.008 | -0.051 | 0.020 |
| 0.6 | 0.2 | 0.6 | 0 | 0 | 0 | 10 | 10 | 0.162 | -0.001 | -0.416 | -0.336 | -0.008 | 0.000 | -0.072 | -0.027 | -0.032 | -0.013 |
| 0.6 | 0.2 | 0 | 0 | 0.6 | 0.2 | 10 | 10 | 0.375 | -0.003 | -0.866 | -0.329 | -0.009 | -0.010 | -0.145 | -0.047 | -0.034 | -0.007 |
| 0.6 | 0.2 | 0 | 0 | 0.6 | 0 | 10 | 10 | 1.881 | -0.004 | -0.913 | -0.331 | -0.031 | -0.045 | -0.132 | -0.033 | -0.020 | 0.030 |
| 0.6 | 0.2 | 0 | 0 | 0 | 0 | 10 | 10 | 0.285 | -0.001 | -0.686 | -0.302 | -0.012 | 0.001 | -0.106 | -0.062 | -0.020 | -0.009 |
| 0.6 | 0 | 0.6 | 0.2 | 0.6 | 0.2 | 10 | 10 | -0.581 | -0.002 | -0.541 | -0.340 | -0.004 | 0.020 | -0.103 | -0.074 | -0.047 | -0.033 |
| 0.6 | 0 | 0.6 | 0.2 | 0.6 | 0 | 10 | 10 | -0.465 | -0.003 | -0.504 | -0.387 | -0.004 | 0.010 | -0.084 | -0.054 | -0.062 | -0.028 |
| 0.6 | 0 | 0.6 | 0.2 | 0 | 0 | 10 | 10 | -0.060 | -0.003 | -0.350 | -0.360 | -0.028 | 0.014 | -0.034 | -0.055 | -0.020 | -0.031 |
| 0.6 | 0 | 0.6 | 0 | 0.6 | 0.2 | 10 | 10 | -0.160 | -0.002 | -0.654 | -0.329 | -0.006 | 0.007 | -0.141 | -0.064 | -0.043 | -0.020 |
| 0.6 | 0 | 0.6 | 0 | 0.6 | 0 | 10 | 10 | -0.002 | -0.002 | -0.615 | -0.367 | -0.004 | -0.007 | -0.126 | -0.040 | -0.058 | -0.009 |
| 0.6 | 0 | 0.6 | 0 | 0 | 0 | 10 | 10 | 0.215 | -0.003 | -0.404 | -0.354 | -0.022 | -0.001 | -0.067 | -0.032 | -0.022 | -0.014 |
| 0.6 | 0 | 0 | 0 | 0.6 | 0.2 | 10 | 10 | 0.093 | -0.003 | -0.869 | -0.332 | -0.024 | 0.012 | -0.135 | -0.076 | -0.025 | -0.027 |
| 0.6 | 0 | 0 | 0 | 0.6 | 0 | 10 | 10 | 0.314 | -0.003 | -0.945 | -0.338 | -0.022 | -0.007 | -0.159 | -0.072 | -0.034 | -0.007 |
| 0.6 | 0 | 0 | 0 | 0 | 0 | 10 | 10 | 0.270 | -0.002 | -0.685 | -0.330 | -0.024 | 0.002 | -0.103 | -0.061 | -0.013 | -0.011 |
| 0 | 0 | 0.6 | 0.2 | 0.6 | 0.2 | 10 | 10 | -0.397 | -0.001 | -0.516 | -0.308 | 0.019 | 0.018 | -0.110 | -0.074 | -0.052 | -0.025 |
| 0 | 0 | 0.6 | 0.2 | 0.6 | 0 | 10 | 10 | -0.393 | -0.002 | -0.442 | -0.356 | 0.019 | 0.016 | -0.090 | -0.059 | -0.067 | -0.028 |
| 0 | 0 | 0.6 | 0.2 | 0 | 0 | 10 | 10 | -0.123 | -0.006 | -0.439 | -0.452 | -0.020 | 0.010 | -0.044 | -0.052 | -0.036 | -0.035 |
| 0 | 0 | 0.6 | 0 | 0.6 | 0.2 | 10 | 10 | -0.098 | -0.002 | -0.644 | -0.328 | 0.006 | 0.004 | -0.144 | -0.067 | -0.044 | -0.016 |
| 0 | 0 | 0.6 | 0 | 0.6 | 0 | 10 | 10 | -0.003 | -0.004 | -0.604 | -0.385 | -0.001 | -0.010 | -0.123 | -0.051 | -0.055 | -0.012 |
| 0 | 0 | 0.6 | 0 | 0 | 0 | 10 | 10 | 0.146 | -0.006 | -0.530 | -0.455 | -0.029 | -0.011 | -0.069 | -0.037 | -0.028 | -0.016 |
| 0 | 0 | 0 | 0 | 0.6 | 0.2 | 10 | 10 | 0.082 | -0.004 | -0.863 | -0.357 | -0.019 | 0.001 | -0.131 | -0.071 | -0.026 | -0.020 |
| 0 | 0 | 0 | 0 | 0.6 | 0 | 10 | 10 | 0.229 | -0.006 | -0.981 | -0.412 | -0.031 | -0.016 | -0.151 | -0.080 | -0.031 | -0.012 |
| 0 | 0 | 0 | 0 | 0 | 0 | 10 | 10 | 0.333 | -0.007 | -0.816 | -0.461 | -0.047 | -0.020 | -0.092 | -0.064 | -0.008 | -0.005 |

Table 6: Bias of Parameters, $N = 100$, $T = 10$, $\theta = 0.5$

| **Configuration** | | | | | | | | **Bias** | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\rho_1^L$ | $\rho_2^L$ | $\rho_1^A$ | $\rho_2^A$ | $\rho_1^B$ | $\rho_2^B$ | $\sigma_\mu^2$ | $\sigma_\nu^2$ | $\beta_0$ | $\beta_1$ | $\sigma_\mu^2$ | $\sigma_\nu^2$ | $\rho_1^L$ | $\rho_2^L$ | $\rho_1^A$ | $\rho_2^A$ | $\rho_1^B$ | $\rho_2^B$ |
| 0.6 | 0.2 | 0.6 | 0.2 | 0.6 | 0.2 | 10 | 10 | -0.324 | -0.004 | -0.427 | -0.177 | 0.003 | -0.004 | -0.077 | -0.039 | -0.029 | -0.009 |
| 0.6 | 0.2 | 0.6 | 0.2 | 0.6 | 0 | 10 | 10 | -0.139 | -0.003 | -0.391 | -0.174 | 0.001 | -0.010 | -0.060 | -0.030 | -0.030 | 0.001 |
| 0.6 | 0.2 | 0.6 | 0.2 | 0 | 0 | 10 | 10 | 0.018 | -0.003 | -0.311 | -0.155 | -0.006 | -0.001 | -0.040 | -0.033 | -0.010 | -0.004 |
| 0.6 | 0.2 | 0.6 | 0 | 0.6 | 0.2 | 10 | 10 | 0.186 | -0.004 | -0.531 | -0.172 | -0.001 | -0.013 | -0.105 | -0.032 | -0.025 | 0.001 |
| 0.6 | 0.2 | 0.6 | 0 | 0.6 | 0 | 10 | 10 | 0.101 | -0.003 | -0.459 | -0.170 | 0.004 | -0.014 | -0.091 | -0.026 | -0.031 | 0.005 |
| 0.6 | 0.2 | 0.6 | 0 | 0 | 0 | 10 | 10 | 0.224 | -0.002 | -0.360 | -0.139 | -0.005 | -0.004 | -0.060 | -0.025 | -0.006 | 0.003 |
| 0.6 | 0.2 | 0 | 0 | 0.6 | 0.2 | 10 | 10 | 0.327 | -0.004 | -0.795 | -0.160 | -0.007 | -0.009 | -0.139 | -0.055 | -0.016 | -0.003 |
| 0.6 | 0.2 | 0 | 0 | 0.6 | 0 | 10 | 10 | 0.514 | -0.004 | -0.748 | -0.161 | -0.004 | -0.016 | -0.139 | -0.051 | -0.019 | 0.010 |
| 0.6 | 0.2 | 0 | 0 | 0 | 0 | 10 | 10 | 0.282 | -0.002 | -0.581 | -0.123 | -0.006 | -0.005 | -0.098 | -0.049 | -0.003 | 0.006 |
| 0.6 | 0 | 0.6 | 0.2 | 0.6 | 0.2 | 10 | 10 | -0.285 | -0.003 | -0.404 | -0.196 | -0.006 | 0.015 | -0.076 | -0.059 | -0.024 | -0.025 |
| 0.6 | 0 | 0.6 | 0.2 | 0.6 | 0 | 10 | 10 | -0.275 | -0.004 | -0.383 | -0.224 | -0.006 | 0.008 | -0.066 | -0.045 | -0.031 | -0.019 |
| 0.6 | 0 | 0.6 | 0.2 | 0 | 0 | 10 | 10 | -0.044 | -0.003 | -0.297 | -0.167 | -0.013 | 0.005 | -0.038 | -0.039 | -0.008 | -0.012 |
| 0.6 | 0 | 0.6 | 0 | 0.6 | 0.2 | 10 | 10 | -0.035 | -0.004 | -0.509 | -0.197 | -0.009 | 0.007 | -0.105 | -0.053 | -0.020 | -0.017 |
| 0.6 | 0 | 0.6 | 0 | 0.6 | 0 | 10 | 10 | 0.114 | -0.004 | -0.488 | -0.230 | -0.011 | -0.005 | -0.094 | -0.037 | -0.027 | -0.006 |
| 0.6 | 0 | 0.6 | 0 | 0 | 0 | 10 | 10 | 0.193 | -0.003 | -0.379 | -0.159 | -0.014 | -0.004 | -0.058 | -0.028 | -0.004 | -0.001 |
| 0.6 | 0 | 0 | 0 | 0.6 | 0.2 | 10 | 10 | 0.113 | -0.004 | -0.775 | -0.181 | -0.015 | 0.005 | -0.136 | -0.072 | -0.012 | -0.016 |
| 0.6 | 0 | 0 | 0 | 0.6 | 0 | 10 | 10 | 0.365 | -0.004 | -0.804 | -0.207 | -0.019 | -0.011 | -0.139 | -0.061 | -0.015 | 0.002 |
| 0.6 | 0 | 0 | 0 | 0 | 0 | 10 | 10 | 0.258 | -0.003 | -0.615 | -0.148 | -0.014 | -0.006 | -0.095 | -0.050 | 0.000 | 0.004 |
| 0 | 0 | 0.6 | 0.2 | 0.6 | 0.2 | 10 | 10 | -0.264 | -0.002 | -0.356 | -0.131 | 0.021 | 0.012 | -0.079 | -0.047 | -0.031 | -0.009 |
| 0 | 0 | 0.6 | 0.2 | 0.6 | 0 | 10 | 10 | -0.299 | -0.002 | -0.351 | -0.168 | 0.021 | 0.015 | -0.075 | -0.046 | -0.041 | -0.020 |
| 0 | 0 | 0.6 | 0.2 | 0 | 0 | 10 | 10 | -0.073 | -0.005 | -0.345 | -0.225 | -0.008 | 0.005 | -0.044 | -0.039 | -0.016 | -0.015 |
| 0 | 0 | 0.6 | 0 | 0.6 | 0.2 | 10 | 10 | -0.120 | -0.002 | -0.485 | -0.151 | 0.013 | 0.008 | -0.110 | -0.052 | -0.027 | -0.009 |
| 0 | 0 | 0.6 | 0 | 0.6 | 0 | 10 | 10 | -0.082 | -0.003 | -0.472 | -0.192 | 0.008 | 0.004 | -0.100 | -0.048 | -0.034 | -0.013 |
| 0 | 0 | 0.6 | 0 | 0 | 0 | 10 | 10 | 0.091 | -0.005 | -0.432 | -0.231 | -0.014 | -0.007 | -0.063 | -0.031 | -0.012 | -0.004 |
| 0 | 0 | 0 | 0 | 0.6 | 0.2 | 10 | 10 | -0.015 | -0.003 | -0.771 | -0.162 | 0.001 | 0.001 | -0.141 | -0.068 | -0.018 | -0.006 |
| 0 | 0 | 0 | 0 | 0.6 | 0 | 10 | 10 | 0.078 | -0.004 | -0.825 | -0.207 | -0.009 | -0.006 | -0.146 | -0.074 | -0.021 | -0.007 |
| 0 | 0 | 0 | 0 | 0 | 0 | 10 | 10 | 0.152 | -0.005 | -0.680 | -0.230 | -0.017 | -0.013 | -0.100 | -0.052 | -0.008 | 0.002 |

Table 7: Bias of Parameters, $N = 200$, $T = 5$, $\theta = 0.5$

| **Configuration** | | | | | | | | **Bias** | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\rho_1^L$ | $\rho_2^L$ | $\rho_1^A$ | $\rho_2^A$ | $\rho_1^B$ | $\rho_2^B$ | $\sigma_\mu^2$ | $\sigma_\nu^2$ | $\beta_0$ | $\beta_1$ | $\sigma_\mu^2$ | $\sigma_\nu^2$ | $\rho_1^L$ | $\rho_2^L$ | $\rho_1^A$ | $\rho_2^A$ | $\rho_1^B$ | $\rho_2^B$ |
| 0.6 | 0.2 | 0.6 | 0.2 | 0.6 | 0.2 | 10 | 10 | -0.258 | -0.003 | -0.316 | -0.166 | 0.001 | 0.000 | -0.048 | -0.029 | -0.025 | -0.015 |
| 0.6 | 0.2 | 0.6 | 0.2 | 0.6 | 0 | 10 | 10 | -0.047 | -0.003 | -0.259 | -0.167 | -0.001 | -0.005 | -0.036 | -0.014 | -0.024 | -0.005 |
| 0.6 | 0.2 | 0.6 | 0.2 | 0 | 0 | 10 | 10 | 0.127 | -0.003 | -0.212 | -0.128 | -0.009 | 0.001 | -0.014 | -0.017 | -0.005 | -0.006 |
| 0.6 | 0.2 | 0.6 | 0 | 0.6 | 0.2 | 10 | 10 | 0.074 | -0.004 | -0.379 | -0.157 | 0.002 | -0.009 | -0.071 | -0.021 | -0.024 | -0.004 |
| 0.6 | 0.2 | 0.6 | 0 | 0.6 | 0 | 10 | 10 | 0.134 | -0.002 | -0.321 | -0.142 | 0.003 | -0.011 | -0.058 | -0.009 | -0.026 | 0.002 |
| 0.6 | 0.2 | 0.6 | 0 | 0 | 0 | 10 | 10 | 0.233 | -0.002 | -0.249 | -0.121 | -0.008 | -0.002 | -0.026 | -0.010 | -0.004 | -0.001 |
| 0.6 | 0.2 | 0 | 0 | 0.6 | 0.2 | 10 | 10 | 0.148 | -0.003 | -0.575 | -0.142 | -0.002 | -0.006 | -0.099 | -0.038 | -0.018 | -0.008 |
| 0.6 | 0.2 | 0 | 0 | 0.6 | 0 | 10 | 10 | 0.560 | -0.003 | -0.587 | -0.125 | -0.003 | -0.020 | -0.105 | -0.031 | -0.016 | 0.011 |
| 0.6 | 0.2 | 0 | 0 | 0 | 0 | 10 | 10 | 0.261 | -0.002 | -0.373 | -0.107 | -0.007 | -0.003 | -0.048 | -0.027 | -0.001 | 0.001 |
| 0.6 | 0 | 0.6 | 0.2 | 0.6 | 0.2 | 10 | 10 | -0.312 | -0.003 | -0.334 | -0.170 | -0.002 | 0.019 | -0.050 | -0.048 | -0.025 | -0.031 |
| 0.6 | 0 | 0.6 | 0.2 | 0.6 | 0 | 10 | 10 | -0.194 | -0.004 | -0.296 | -0.204 | -0.007 | 0.010 | -0.036 | -0.033 | -0.029 | -0.025 |
| 0.6 | 0 | 0.6 | 0.2 | 0 | 0 | 10 | 10 | 0.057 | -0.003 | -0.219 | -0.147 | -0.017 | 0.006 | -0.011 | -0.025 | -0.004 | -0.017 |
| 0.6 | 0 | 0.6 | 0 | 0.6 | 0.2 | 10 | 10 | -0.077 | -0.003 | -0.398 | -0.170 | -0.007 | 0.011 | -0.070 | -0.044 | -0.020 | -0.024 |
| 0.6 | 0 | 0.6 | 0 | 0.6 | 0 | 10 | 10 | 0.098 | -0.004 | -0.353 | -0.192 | -0.010 | -0.002 | -0.057 | -0.024 | -0.024 | -0.011 |
| 0.6 | 0 | 0.6 | 0 | 0 | 0 | 10 | 10 | 0.219 | -0.003 | -0.258 | -0.135 | -0.018 | -0.001 | -0.023 | -0.015 | 0.001 | -0.007 |
| 0.6 | 0 | 0 | 0 | 0.6 | 0.2 | 10 | 10 | -0.012 | -0.003 | -0.581 | -0.152 | -0.011 | 0.011 | -0.096 | -0.058 | -0.014 | -0.025 |
| 0.6 | 0 | 0 | 0 | 0.6 | 0 | 10 | 10 | 0.240 | -0.004 | -0.621 | -0.168 | -0.014 | -0.006 | -0.105 | -0.051 | -0.015 | -0.006 |
| 0.6 | 0 | 0 | 0 | 0 | 0 | 10 | 10 | 0.227 | -0.002 | -0.393 | -0.125 | -0.015 | -0.003 | -0.046 | -0.029 | 0.000 | -0.003 |
| 0 | 0 | 0.6 | 0.2 | 0.6 | 0.2 | 10 | 10 | -0.152 | -0.002 | -0.272 | -0.121 | 0.012 | 0.012 | -0.047 | -0.032 | -0.023 | -0.014 |
| 0 | 0 | 0.6 | 0.2 | 0.6 | 0 | 10 | 10 | -0.149 | -0.002 | -0.260 | -0.161 | 0.010 | 0.013 | -0.039 | -0.030 | -0.032 | -0.022 |
| 0 | 0 | 0.6 | 0.2 | 0 | 0 | 10 | 10 | 0.009 | -0.005 | -0.272 | -0.205 | -0.013 | 0.006 | -0.016 | -0.025 | -0.013 | -0.020 |
| 0 | 0 | 0.6 | 0 | 0.6 | 0.2 | 10 | 10 | -0.072 | -0.003 | -0.366 | -0.134 | 0.006 | 0.008 | -0.069 | -0.036 | -0.020 | -0.012 |
| 0 | 0 | 0.6 | 0 | 0.6 | 0 | 10 | 10 | -0.023 | -0.004 | -0.361 | -0.182 | -0.001 | 0.004 | -0.059 | -0.032 | -0.027 | -0.017 |
| 0 | 0 | 0.6 | 0 | 0 | 0 | 10 | 10 | 0.095 | -0.005 | -0.324 | -0.214 | -0.017 | -0.004 | -0.029 | -0.018 | -0.010 | -0.011 |
| 0 | 0 | 0 | 0 | 0.6 | 0.2 | 10 | 10 | -0.045 | -0.003 | -0.584 | -0.131 | 0.002 | 0.007 | -0.100 | -0.052 | -0.016 | -0.013 |
| 0 | 0 | 0 | 0 | 0.6 | 0 | 10 | 10 | 0.021 | -0.004 | -0.649 | -0.174 | -0.006 | 0.001 | -0.110 | -0.061 | -0.020 | -0.015 |
| 0 | 0 | 0 | 0 | 0 | 0 | 10 | 10 | 0.130 | -0.005 | -0.481 | -0.205 | -0.019 | -0.008 | -0.052 | -0.032 | -0.006 | -0.006 |

Table 8: Bias of Parameters, $N = 200$, $T = 10$, $\theta = 0.5$

| **Configuration** | | | | | | | | **Bias** | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\rho_1^L$ | $\rho_2^L$ | $\rho_1^A$ | $\rho_2^A$ | $\rho_1^B$ | $\rho_2^B$ | $\sigma_\mu^2$ | $\sigma_\nu^2$ | $\beta_0$ | $\beta_1$ | $\sigma_\mu^2$ | $\sigma_\nu^2$ | $\rho_1^L$ | $\rho_2^L$ | $\rho_1^A$ | $\rho_2^A$ | $\rho_1^B$ | $\rho_2^B$ |
| 0.6 | 0.2 | 0.6 | 0.2 | 0.6 | 0.2 | 10 | 10 | -0.038 | -0.001 | -0.248 | -0.082 | -0.009 | 0.004 | -0.026 | -0.028 | -0.007 | -0.013 |
| 0.6 | 0.2 | 0.6 | 0.2 | 0.6 | 0 | 10 | 10 | 0.135 | 0.000 | -0.237 | -0.077 | -0.008 | -0.004 | -0.021 | -0.015 | -0.006 | 0.000 |
| 0.6 | 0.2 | 0.6 | 0.2 | 0 | 0 | 10 | 10 | 0.130 | 0.000 | -0.173 | -0.038 | -0.009 | 0.002 | -0.011 | -0.017 | 0.006 | -0.002 |
| 0.6 | 0.2 | 0.6 | 0 | 0.6 | 0.2 | 10 | 10 | 0.338 | -0.001 | -0.290 | -0.084 | -0.013 | -0.004 | -0.036 | -0.019 | -0.003 | -0.005 |
| 0.6 | 0.2 | 0.6 | 0 | 0.6 | 0 | 10 | 10 | 0.294 | 0.000 | -0.266 | -0.074 | -0.007 | -0.007 | -0.036 | -0.009 | -0.006 | 0.004 |
| 0.6 | 0.2 | 0.6 | 0 | 0 | 0 | 10 | 10 | 0.196 | 0.000 | -0.198 | -0.034 | -0.008 | 0.000 | -0.021 | -0.012 | 0.007 | 0.002 |
| 0.6 | 0.2 | 0 | 0 | 0.6 | 0.2 | 10 | 10 | 0.463 | -0.001 | -0.470 | -0.080 | -0.015 | -0.005 | -0.066 | -0.033 | 0.001 | -0.004 |
| 0.6 | 0.2 | 0 | 0 | 0.6 | 0 | 10 | 10 | 0.413 | 0.000 | -0.448 | -0.064 | -0.008 | -0.009 | -0.069 | -0.027 | -0.003 | 0.008 |
| 0.6 | 0.2 | 0 | 0 | 0 | 0 | 10 | 10 | 0.222 | 0.000 | -0.311 | -0.029 | -0.008 | -0.001 | -0.037 | -0.022 | 0.008 | 0.004 |
| 0.6 | 0 | 0.6 | 0.2 | 0.6 | 0.2 | 10 | 10 | -0.213 | 0.000 | -0.236 | -0.080 | -0.007 | 0.019 | -0.030 | -0.040 | -0.008 | -0.026 |
| 0.6 | 0 | 0.6 | 0.2 | 0.6 | 0 | 10 | 10 | 0.022 | -0.001 | -0.250 | -0.112 | -0.017 | 0.006 | -0.018 | -0.028 | -0.006 | -0.015 |
| 0.6 | 0 | 0.6 | 0.2 | 0 | 0 | 10 | 10 | 0.089 | 0.000 | -0.183 | -0.048 | -0.013 | 0.002 | -0.010 | -0.020 | 0.005 | -0.004 |
| 0.6 | 0 | 0.6 | 0 | 0.6 | 0.2 | 10 | 10 | -0.005 | -0.001 | -0.305 | -0.086 | -0.014 | 0.012 | -0.041 | -0.037 | -0.003 | -0.020 |
| 0.6 | 0 | 0.6 | 0 | 0.6 | 0 | 10 | 10 | 0.241 | -0.002 | -0.304 | -0.113 | -0.020 | -0.002 | -0.031 | -0.019 | -0.003 | -0.006 |
| 0.6 | 0 | 0.6 | 0 | 0 | 0 | 10 | 10 | 0.172 | 0.000 | -0.213 | -0.045 | -0.013 | -0.002 | -0.020 | -0.013 | 0.007 | 0.001 |
| 0.6 | 0 | 0 | 0 | 0.6 | 0.2 | 10 | 10 | 0.026 | -0.001 | -0.481 | -0.078 | -0.016 | 0.012 | -0.073 | -0.054 | 0.001 | -0.020 |
| 0.6 | 0 | 0 | 0 | 0.6 | 0 | 10 | 10 | 0.366 | -0.002 | -0.471 | -0.101 | -0.024 | -0.007 | -0.061 | -0.033 | 0.003 | -0.001 |
| 0.6 | 0 | 0 | 0 | 0 | 0 | 10 | 10 | 0.203 | 0.000 | -0.323 | -0.042 | -0.013 | -0.004 | -0.036 | -0.021 | 0.009 | 0.004 |
| 0 | 0 | 0.6 | 0.2 | 0.6 | 0.2 | 10 | 10 | -0.072 | 0.000 | -0.226 | -0.070 | -0.002 | 0.008 | -0.025 | -0.024 | -0.006 | -0.009 |
| 0 | 0 | 0.6 | 0.2 | 0.6 | 0 | 10 | 10 | -0.070 | -0.001 | -0.225 | -0.092 | -0.002 | 0.009 | -0.024 | -0.025 | -0.012 | -0.014 |
| 0 | 0 | 0.6 | 0.2 | 0 | 0 | 10 | 10 | 0.026 | -0.002 | -0.226 | -0.099 | -0.015 | 0.005 | -0.011 | -0.022 | 0.002 | -0.011 |
| 0 | 0 | 0.6 | 0 | 0.6 | 0.2 | 10 | 10 | -0.031 | -0.001 | -0.280 | -0.073 | -0.004 | 0.005 | -0.041 | -0.026 | -0.005 | -0.007 |
| 0 | 0 | 0.6 | 0 | 0.6 | 0 | 10 | 10 | -0.016 | -0.001 | -0.284 | -0.100 | -0.006 | 0.005 | -0.038 | -0.026 | -0.010 | -0.012 |
| 0 | 0 | 0.6 | 0 | 0 | 0 | 10 | 10 | 0.074 | -0.002 | -0.263 | -0.102 | -0.017 | 0.000 | -0.021 | -0.018 | 0.003 | -0.006 |
| 0 | 0 | 0 | 0 | 0.6 | 0.2 | 10 | 10 | -0.016 | -0.001 | -0.489 | -0.072 | -0.006 | 0.005 | -0.079 | -0.047 | -0.004 | -0.008 |
| 0 | 0 | 0 | 0 | 0.6 | 0 | 10 | 10 | 0.025 | -0.001 | -0.503 | -0.098 | -0.010 | 0.002 | -0.078 | -0.048 | -0.007 | -0.009 |
| 0 | 0 | 0 | 0 | 0 | 0 | 10 | 10 | 0.091 | -0.001 | -0.381 | -0.099 | -0.017 | -0.003 | -0.040 | -0.028 | 0.004 | -0.003 |

Table 9: RMSE of Parameters, $N = 100$, $T = 5$, $\theta = 0.25$

| **Configuration** | | | | | | | | **RMSE** | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\rho_1^L$ | $\rho_2^L$ | $\rho_1^A$ | $\rho_2^A$ | $\rho_1^B$ | $\rho_2^B$ | $\sigma_\mu^2$ | $\sigma_\nu^2$ | $\beta_0$ | $\beta_1$ | $\sigma_\mu^2$ | $\sigma_\nu^2$ | $\rho_1^L$ | $\rho_2^L$ | $\rho_1^A$ | $\rho_2^A$ | $\rho_1^B$ | $\rho_2^B$ |
| 0.6 | 0.2 | 0.6 | 0.2 | 0.6 | 0.2 | 5 | 15 | 2.360 | 0.045 | 1.266 | 1.217 | 0.162 | 0.136 | 0.399 | 0.280 | 0.197 | 0.153 |
| 0.6 | 0.2 | 0.6 | 0.2 | 0.6 | 0 | 5 | 15 | 2.182 | 0.044 | 1.228 | 1.248 | 0.155 | 0.143 | 0.328 | 0.258 | 0.212 | 0.152 |
| 0.6 | 0.2 | 0.6 | 0.2 | 0 | 0 | 5 | 15 | 1.966 | 0.046 | 1.287 | 1.362 | 0.136 | 0.123 | 0.226 | 0.215 | 0.243 | 0.152 |
| 0.6 | 0.2 | 0.6 | 0 | 0.6 | 0.2 | 5 | 15 | 2.367 | 0.045 | 1.287 | 1.216 | 0.159 | 0.139 | 0.450 | 0.284 | 0.194 | 0.156 |
| 0.6 | 0.2 | 0.6 | 0 | 0.6 | 0 | 5 | 15 | 2.194 | 0.045 | 1.263 | 1.234 | 0.151 | 0.144 | 0.414 | 0.271 | 0.206 | 0.156 |
| 0.6 | 0.2 | 0.6 | 0 | 0 | 0 | 5 | 15 | 1.769 | 0.047 | 1.250 | 1.359 | 0.132 | 0.120 | 0.256 | 0.209 | 0.238 | 0.151 |
| 0.6 | 0.2 | 0 | 0 | 0.6 | 0.2 | 5 | 15 | 2.317 | 0.045 | 1.362 | 1.219 | 0.154 | 0.141 | 0.460 | 0.336 | 0.184 | 0.157 |
| 0.6 | 0.2 | 0 | 0 | 0.6 | 0 | 5 | 15 | 2.108 | 0.046 | 1.375 | 1.222 | 0.149 | 0.146 | 0.470 | 0.321 | 0.194 | 0.155 |
| 0.6 | 0.2 | 0 | 0 | 0 | 0 | 5 | 15 | 1.636 | 0.047 | 1.260 | 1.295 | 0.129 | 0.120 | 0.387 | 0.295 | 0.224 | 0.149 |
| 0.6 | 0 | 0.6 | 0.2 | 0.6 | 0.2 | 5 | 15 | 2.356 | 0.046 | 1.259 | 1.238 | 0.182 | 0.132 | 0.408 | 0.292 | 0.197 | 0.155 |
| 0.6 | 0 | 0.6 | 0.2 | 0.6 | 0 | 5 | 15 | 2.196 | 0.045 | 1.238 | 1.267 | 0.181 | 0.138 | 0.350 | 0.266 | 0.218 | 0.159 |
| 0.6 | 0 | 0.6 | 0.2 | 0 | 0 | 5 | 15 | 1.935 | 0.047 | 1.262 | 1.361 | 0.152 | 0.134 | 0.225 | 0.216 | 0.252 | 0.185 |
| 0.6 | 0 | 0.6 | 0 | 0.6 | 0.2 | 5 | 15 | 2.248 | 0.046 | 1.292 | 1.230 | 0.179 | 0.129 | 0.459 | 0.286 | 0.194 | 0.152 |
| 0.6 | 0 | 0.6 | 0 | 0.6 | 0 | 5 | 15 | 2.194 | 0.045 | 1.265 | 1.245 | 0.176 | 0.137 | 0.409 | 0.269 | 0.210 | 0.156 |
| 0.6 | 0 | 0.6 | 0 | 0 | 0 | 5 | 15 | 1.849 | 0.047 | 1.243 | 1.358 | 0.148 | 0.130 | 0.258 | 0.209 | 0.248 | 0.181 |
| 0.6 | 0 | 0 | 0 | 0.6 | 0.2 | 5 | 15 | 2.238 | 0.046 | 1.347 | 1.221 | 0.179 | 0.138 | 0.469 | 0.337 | 0.183 | 0.154 |
| 0.6 | 0 | 0 | 0 | 0.6 | 0 | 5 | 15 | 2.155 | 0.046 | 1.383 | 1.228 | 0.171 | 0.140 | 0.466 | 0.325 | 0.194 | 0.156 |
| 0.6 | 0 | 0 | 0 | 0 | 0 | 5 | 15 | 1.702 | 0.047 | 1.256 | 1.304 | 0.144 | 0.128 | 0.391 | 0.296 | 0.230 | 0.175 |
| 0 | 0 | 0.6 | 0.2 | 0.6 | 0.2 | 5 | 15 | 2.207 | 0.049 | 1.272 | 1.486 | 0.294 | 0.157 | 0.420 | 0.297 | 0.201 | 0.145 |
| 0 | 0 | 0.6 | 0.2 | 0.6 | 0 | 5 | 15 | 2.137 | 0.049 | 1.255 | 1.497 | 0.295 | 0.180 | 0.355 | 0.270 | 0.225 | 0.156 |
| 0 | 0 | 0.6 | 0.2 | 0 | 0 | 5 | 15 | 1.883 | 0.048 | 1.284 | 1.360 | 0.251 | 0.186 | 0.233 | 0.220 | 0.271 | 0.208 |
| 0 | 0 | 0.6 | 0 | 0.6 | 0.2 | 5 | 15 | 2.005 | 0.049 | 1.300 | 1.469 | 0.286 | 0.155 | 0.459 | 0.290 | 0.194 | 0.145 |
| 0 | 0 | 0.6 | 0 | 0.6 | 0 | 5 | 15 | 2.025 | 0.049 | 1.265 | 1.492 | 0.293 | 0.177 | 0.425 | 0.273 | 0.219 | 0.154 |
| 0 | 0 | 0.6 | 0 | 0 | 0 | 5 | 15 | 1.784 | 0.048 | 1.259 | 1.355 | 0.253 | 0.188 | 0.267 | 0.215 | 0.268 | 0.207 |
| 0 | 0 | 0 | 0 | 0.6 | 0.2 | 5 | 15 | 1.944 | 0.049 | 1.344 | 1.497 | 0.281 | 0.154 | 0.474 | 0.337 | 0.182 | 0.142 |
| 0 | 0 | 0 | 0 | 0.6 | 0 | 5 | 15 | 1.939 | 0.049 | 1.362 | 1.505 | 0.285 | 0.177 | 0.470 | 0.327 | 0.201 | 0.149 |
| 0 | 0 | 0 | 0 | 0 | 0 | 5 | 15 | 1.680 | 0.048 | 1.246 | 1.319 | 0.244 | 0.188 | 0.399 | 0.303 | 0.249 | 0.202 |

Table 10: RMSE of Parameters, $N = 100$, $T = 10$, $\theta = 0.25$

| **Configuration** | | | | | | | | **RMSE** | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\rho_1^L$ | $\rho_2^L$ | $\rho_1^A$ | $\rho_2^A$ | $\rho_1^B$ | $\rho_2^B$ | $\sigma_\mu^2$ | $\sigma_\nu^2$ | $\beta_0$ | $\beta_1$ | $\sigma_\mu^2$ | $\sigma_\nu^2$ | $\rho_1^L$ | $\rho_2^L$ | $\rho_1^A$ | $\rho_2^A$ | $\rho_1^B$ | $\rho_2^B$ |
| 0.6 | 0.2 | 0.6 | 0.2 | 0.6 | 0.2 | 5 | 15 | 2.302 | 0.034 | 0.988 | 0.814 | 0.135 | 0.116 | 0.286 | 0.226 | 0.151 | 0.124 |
| 0.6 | 0.2 | 0.6 | 0.2 | 0.6 | 0 | 5 | 15 | 2.053 | 0.034 | 0.960 | 0.817 | 0.128 | 0.115 | 0.234 | 0.201 | 0.157 | 0.116 |
| 0.6 | 0.2 | 0.6 | 0.2 | 0 | 0 | 5 | 15 | 1.680 | 0.035 | 0.978 | 0.884 | 0.106 | 0.094 | 0.185 | 0.178 | 0.179 | 0.112 |
| 0.6 | 0.2 | 0.6 | 0 | 0.6 | 0.2 | 5 | 15 | 2.283 | 0.035 | 1.019 | 0.804 | 0.134 | 0.118 | 0.329 | 0.231 | 0.147 | 0.126 |
| 0.6 | 0.2 | 0.6 | 0 | 0.6 | 0 | 5 | 15 | 2.027 | 0.035 | 0.993 | 0.801 | 0.127 | 0.119 | 0.294 | 0.213 | 0.155 | 0.123 |
| 0.6 | 0.2 | 0.6 | 0 | 0 | 0 | 5 | 15 | 1.478 | 0.035 | 0.971 | 0.886 | 0.104 | 0.092 | 0.205 | 0.174 | 0.178 | 0.110 |
| 0.6 | 0.2 | 0 | 0 | 0.6 | 0.2 | 5 | 15 | 2.230 | 0.035 | 1.131 | 0.787 | 0.124 | 0.115 | 0.422 | 0.282 | 0.135 | 0.124 |
| 0.6 | 0.2 | 0 | 0 | 0.6 | 0 | 5 | 15 | 1.969 | 0.035 | 1.144 | 0.790 | 0.120 | 0.116 | 0.419 | 0.275 | 0.141 | 0.121 |
| 0.6 | 0.2 | 0 | 0 | 0 | 0 | 5 | 15 | 1.407 | 0.036 | 0.998 | 0.870 | 0.101 | 0.090 | 0.328 | 0.244 | 0.170 | 0.110 |
| 0.6 | 0 | 0.6 | 0.2 | 0.6 | 0.2 | 5 | 15 | 2.224 | 0.034 | 0.977 | 0.817 | 0.151 | 0.115 | 0.284 | 0.231 | 0.150 | 0.126 |
| 0.6 | 0 | 0.6 | 0.2 | 0.6 | 0 | 5 | 15 | 2.099 | 0.034 | 0.961 | 0.833 | 0.150 | 0.122 | 0.239 | 0.211 | 0.166 | 0.130 |
| 0.6 | 0 | 0.6 | 0.2 | 0 | 0 | 5 | 15 | 1.717 | 0.035 | 0.982 | 0.893 | 0.120 | 0.102 | 0.186 | 0.183 | 0.187 | 0.135 |
| 0.6 | 0 | 0.6 | 0 | 0.6 | 0.2 | 5 | 15 | 2.047 | 0.034 | 1.011 | 0.813 | 0.148 | 0.116 | 0.338 | 0.238 | 0.146 | 0.126 |
| 0.6 | 0 | 0.6 | 0 | 0.6 | 0 | 5 | 15 | 2.042 | 0.034 | 0.992 | 0.823 | 0.151 | 0.122 | 0.302 | 0.223 | 0.163 | 0.129 |
| 0.6 | 0 | 0.6 | 0 | 0 | 0 | 5 | 15 | 1.571 | 0.035 | 0.976 | 0.890 | 0.117 | 0.101 | 0.206 | 0.181 | 0.184 | 0.136 |
| 0.6 | 0 | 0 | 0 | 0.6 | 0.2 | 5 | 15 | 1.938 | 0.035 | 1.126 | 0.792 | 0.141 | 0.113 | 0.424 | 0.285 | 0.133 | 0.123 |
| 0.6 | 0 | 0 | 0 | 0.6 | 0 | 5 | 15 | 1.981 | 0.035 | 1.143 | 0.798 | 0.141 | 0.121 | 0.423 | 0.279 | 0.145 | 0.127 |
| 0.6 | 0 | 0 | 0 | 0 | 0 | 5 | 15 | 1.466 | 0.036 | 1.007 | 0.863 | 0.112 | 0.099 | 0.332 | 0.252 | 0.174 | 0.135 |
| 0 | 0 | 0.6 | 0.2 | 0.6 | 0.2 | 5 | 15 | 1.927 | 0.037 | 0.994 | 0.963 | 0.229 | 0.133 | 0.289 | 0.228 | 0.148 | 0.117 |
| 0 | 0 | 0.6 | 0.2 | 0.6 | 0 | 5 | 15 | 1.951 | 0.037 | 0.976 | 0.968 | 0.232 | 0.154 | 0.245 | 0.210 | 0.166 | 0.124 |
| 0 | 0 | 0.6 | 0.2 | 0 | 0 | 5 | 15 | 1.679 | 0.035 | 0.995 | 0.838 | 0.187 | 0.154 | 0.188 | 0.186 | 0.193 | 0.161 |
| 0 | 0 | 0.6 | 0 | 0.6 | 0.2 | 5 | 15 | 1.654 | 0.037 | 1.023 | 0.966 | 0.227 | 0.129 | 0.346 | 0.239 | 0.145 | 0.114 |
| 0 | 0 | 0.6 | 0 | 0.6 | 0 | 5 | 15 | 1.724 | 0.037 | 0.997 | 0.985 | 0.231 | 0.152 | 0.312 | 0.227 | 0.161 | 0.122 |
| 0 | 0 | 0.6 | 0 | 0 | 0 | 5 | 15 | 1.461 | 0.036 | 1.001 | 0.848 | 0.191 | 0.159 | 0.212 | 0.188 | 0.192 | 0.163 |
| 0 | 0 | 0 | 0 | 0.6 | 0.2 | 5 | 15 | 1.488 | 0.037 | 1.115 | 0.975 | 0.214 | 0.124 | 0.423 | 0.287 | 0.133 | 0.111 |
| 0 | 0 | 0 | 0 | 0.6 | 0 | 5 | 15 | 1.532 | 0.036 | 1.135 | 0.985 | 0.215 | 0.146 | 0.421 | 0.284 | 0.144 | 0.117 |
| 0 | 0 | 0 | 0 | 0 | 0 | 5 | 15 | 1.309 | 0.035 | 1.015 | 0.826 | 0.181 | 0.155 | 0.336 | 0.259 | 0.181 | 0.159 |

Table 11: RMSE of Parameters, $N = 200$, $T = 5$, $\theta = 0.25$

| **Configuration** | | | | | | | | **RMSE** | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\rho_1^L$ | $\rho_2^L$ | $\rho_1^A$ | $\rho_2^A$ | $\rho_1^B$ | $\rho_2^B$ | $\sigma_\mu^2$ | $\sigma_\nu^2$ | $\beta_0$ | $\beta_1$ | $\sigma_\mu^2$ | $\sigma_\nu^2$ | $\rho_1^L$ | $\rho_2^L$ | $\rho_1^A$ | $\rho_2^A$ | $\rho_1^B$ | $\rho_2^B$ |
| 0.6 | 0.2 | 0.6 | 0.2 | 0.6 | 0.2 | 5 | 15 | 2.118 | 0.031 | 0.891 | 0.817 | 0.119 | 0.102 | 0.241 | 0.200 | 0.133 | 0.114 |
| 0.6 | 0.2 | 0.6 | 0.2 | 0.6 | 0 | 5 | 15 | 1.876 | 0.032 | 0.887 | 0.824 | 0.114 | 0.101 | 0.198 | 0.185 | 0.138 | 0.109 |
| 0.6 | 0.2 | 0.6 | 0.2 | 0 | 0 | 5 | 15 | 1.521 | 0.032 | 0.923 | 0.914 | 0.096 | 0.087 | 0.158 | 0.156 | 0.168 | 0.114 |
| 0.6 | 0.2 | 0.6 | 0 | 0.6 | 0.2 | 5 | 15 | 2.114 | 0.031 | 0.921 | 0.814 | 0.116 | 0.105 | 0.292 | 0.211 | 0.131 | 0.117 |
| 0.6 | 0.2 | 0.6 | 0 | 0.6 | 0 | 5 | 15 | 1.890 | 0.032 | 0.894 | 0.810 | 0.110 | 0.104 | 0.244 | 0.198 | 0.135 | 0.113 |
| 0.6 | 0.2 | 0.6 | 0 | 0 | 0 | 5 | 15 | 1.386 | 0.032 | 0.913 | 0.905 | 0.096 | 0.086 | 0.166 | 0.150 | 0.166 | 0.112 |
| 0.6 | 0.2 | 0 | 0 | 0.6 | 0.2 | 5 | 15 | 2.152 | 0.031 | 1.057 | 0.809 | 0.108 | 0.103 | 0.408 | 0.281 | 0.121 | 0.114 |
| 0.6 | 0.2 | 0 | 0 | 0.6 | 0 | 5 | 15 | 1.892 | 0.032 | 1.067 | 0.820 | 0.106 | 0.102 | 0.392 | 0.269 | 0.125 | 0.110 |
| 0.6 | 0.2 | 0 | 0 | 0 | 0 | 5 | 15 | 1.310 | 0.032 | 0.923 | 0.878 | 0.089 | 0.081 | 0.291 | 0.223 | 0.156 | 0.108 |
| 0.6 | 0 | 0.6 | 0.2 | 0.6 | 0.2 | 5 | 15 | 2.122 | 0.031 | 0.902 | 0.830 | 0.138 | 0.106 | 0.254 | 0.209 | 0.135 | 0.121 |
| 0.6 | 0 | 0.6 | 0.2 | 0.6 | 0 | 5 | 15 | 1.987 | 0.031 | 0.881 | 0.847 | 0.137 | 0.112 | 0.208 | 0.187 | 0.150 | 0.125 |
| 0.6 | 0 | 0.6 | 0.2 | 0 | 0 | 5 | 15 | 1.579 | 0.032 | 0.911 | 0.912 | 0.112 | 0.094 | 0.160 | 0.157 | 0.177 | 0.135 |
| 0.6 | 0 | 0.6 | 0 | 0.6 | 0.2 | 5 | 15 | 1.996 | 0.031 | 0.932 | 0.829 | 0.137 | 0.106 | 0.306 | 0.223 | 0.133 | 0.119 |
| 0.6 | 0 | 0.6 | 0 | 0.6 | 0 | 5 | 15 | 1.920 | 0.031 | 0.889 | 0.833 | 0.135 | 0.113 | 0.256 | 0.201 | 0.145 | 0.123 |
| 0.6 | 0 | 0.6 | 0 | 0 | 0 | 5 | 15 | 1.530 | 0.032 | 0.906 | 0.909 | 0.109 | 0.097 | 0.169 | 0.155 | 0.175 | 0.138 |
| 0.6 | 0 | 0 | 0 | 0.6 | 0.2 | 5 | 15 | 1.930 | 0.031 | 1.059 | 0.824 | 0.129 | 0.105 | 0.407 | 0.287 | 0.124 | 0.116 |
| 0.6 | 0 | 0 | 0 | 0.6 | 0 | 5 | 15 | 1.860 | 0.031 | 1.082 | 0.820 | 0.123 | 0.113 | 0.404 | 0.274 | 0.132 | 0.119 |
| 0.6 | 0 | 0 | 0 | 0 | 0 | 5 | 15 | 1.355 | 0.032 | 0.919 | 0.874 | 0.100 | 0.090 | 0.290 | 0.228 | 0.162 | 0.130 |
| 0 | 0 | 0.6 | 0.2 | 0.6 | 0.2 | 5 | 15 | 1.702 | 0.033 | 0.911 | 0.999 | 0.211 | 0.123 | 0.253 | 0.202 | 0.130 | 0.105 |
| 0 | 0 | 0.6 | 0.2 | 0.6 | 0 | 5 | 15 | 1.765 | 0.034 | 0.900 | 1.008 | 0.219 | 0.145 | 0.203 | 0.187 | 0.149 | 0.117 |
| 0 | 0 | 0.6 | 0.2 | 0 | 0 | 5 | 15 | 1.514 | 0.032 | 0.932 | 0.862 | 0.183 | 0.143 | 0.161 | 0.159 | 0.184 | 0.158 |
| 0 | 0 | 0.6 | 0 | 0.6 | 0.2 | 5 | 15 | 1.568 | 0.034 | 0.954 | 1.001 | 0.210 | 0.126 | 0.313 | 0.226 | 0.130 | 0.106 |
| 0 | 0 | 0.6 | 0 | 0.6 | 0 | 5 | 15 | 1.610 | 0.034 | 0.919 | 1.018 | 0.217 | 0.145 | 0.266 | 0.207 | 0.145 | 0.115 |
| 0 | 0 | 0.6 | 0 | 0 | 0 | 5 | 15 | 1.381 | 0.033 | 0.928 | 0.865 | 0.184 | 0.149 | 0.171 | 0.159 | 0.182 | 0.160 |
| 0 | 0 | 0 | 0 | 0.6 | 0.2 | 5 | 15 | 1.470 | 0.033 | 1.057 | 0.995 | 0.199 | 0.123 | 0.412 | 0.291 | 0.122 | 0.102 |
| 0 | 0 | 0 | 0 | 0.6 | 0 | 5 | 15 | 1.474 | 0.033 | 1.081 | 1.007 | 0.203 | 0.142 | 0.401 | 0.284 | 0.134 | 0.111 |
| 0 | 0 | 0 | 0 | 0 | 0 | 5 | 15 | 1.224 | 0.032 | 0.927 | 0.848 | 0.168 | 0.145 | 0.295 | 0.244 | 0.169 | 0.153 |

Table 12: RMSE of Parameters, $N = 200$, $T = 10$, $\theta = 0.25$

| **Configuration** | | | | | | | | **RMSE** | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\rho_1^L$ | $\rho_2^L$ | $\rho_1^A$ | $\rho_2^A$ | $\rho_1^B$ | $\rho_2^B$ | $\sigma_\mu^2$ | $\sigma_\nu^2$ | $\beta_0$ | $\beta_1$ | $\sigma_\mu^2$ | $\sigma_\nu^2$ | $\rho_1^L$ | $\rho_2^L$ | $\rho_1^A$ | $\rho_2^A$ | $\rho_1^B$ | $\rho_2^B$ |
| 0.6 | 0.2 | 0.6 | 0.2 | 0.6 | 0.2 | 5 | 15 | 1.991 | 0.024 | 0.690 | 0.558 | 0.101 | 0.087 | 0.165 | 0.146 | 0.104 | 0.092 |
| 0.6 | 0.2 | 0.6 | 0.2 | 0.6 | 0 | 5 | 15 | 1.771 | 0.024 | 0.685 | 0.558 | 0.093 | 0.082 | 0.153 | 0.138 | 0.105 | 0.083 |
| 0.6 | 0.2 | 0.6 | 0.2 | 0 | 0 | 5 | 15 | 1.288 | 0.025 | 0.705 | 0.621 | 0.075 | 0.064 | 0.128 | 0.122 | 0.125 | 0.078 |
| 0.6 | 0.2 | 0.6 | 0 | 0.6 | 0.2 | 5 | 15 | 2.009 | 0.024 | 0.717 | 0.552 | 0.102 | 0.089 | 0.203 | 0.157 | 0.103 | 0.092 |
| 0.6 | 0.2 | 0.6 | 0 | 0.6 | 0 | 5 | 15 | 1.680 | 0.024 | 0.691 | 0.556 | 0.092 | 0.083 | 0.177 | 0.142 | 0.105 | 0.085 |
| 0.6 | 0.2 | 0.6 | 0 | 0 | 0 | 5 | 15 | 1.108 | 0.025 | 0.704 | 0.619 | 0.074 | 0.064 | 0.137 | 0.119 | 0.124 | 0.078 |
| 0.6 | 0.2 | 0 | 0 | 0.6 | 0.2 | 5 | 15 | 1.965 | 0.024 | 0.870 | 0.556 | 0.097 | 0.087 | 0.352 | 0.238 | 0.097 | 0.090 |
| 0.6 | 0.2 | 0 | 0 | 0.6 | 0 | 5 | 15 | 1.723 | 0.024 | 0.847 | 0.550 | 0.089 | 0.083 | 0.325 | 0.219 | 0.097 | 0.086 |
| 0.6 | 0.2 | 0 | 0 | 0 | 0 | 5 | 15 | 1.015 | 0.025 | 0.699 | 0.610 | 0.071 | 0.061 | 0.229 | 0.174 | 0.120 | 0.076 |
| 0.6 | 0 | 0.6 | 0.2 | 0.6 | 0.2 | 5 | 15 | 1.753 | 0.025 | 0.685 | 0.569 | 0.114 | 0.090 | 0.171 | 0.159 | 0.107 | 0.096 |
| 0.6 | 0 | 0.6 | 0.2 | 0.6 | 0 | 5 | 15 | 1.776 | 0.025 | 0.690 | 0.571 | 0.119 | 0.094 | 0.158 | 0.147 | 0.118 | 0.098 |
| 0.6 | 0 | 0.6 | 0.2 | 0 | 0 | 5 | 15 | 1.358 | 0.025 | 0.708 | 0.619 | 0.085 | 0.072 | 0.129 | 0.124 | 0.130 | 0.099 |
| 0.6 | 0 | 0.6 | 0 | 0.6 | 0.2 | 5 | 15 | 1.596 | 0.025 | 0.707 | 0.574 | 0.115 | 0.088 | 0.210 | 0.169 | 0.106 | 0.095 |
| 0.6 | 0 | 0.6 | 0 | 0.6 | 0 | 5 | 15 | 1.730 | 0.025 | 0.688 | 0.575 | 0.122 | 0.095 | 0.183 | 0.151 | 0.118 | 0.098 |
| 0.6 | 0 | 0.6 | 0 | 0 | 0 | 5 | 15 | 1.225 | 0.025 | 0.706 | 0.617 | 0.083 | 0.073 | 0.138 | 0.123 | 0.129 | 0.101 |
| 0.6 | 0 | 0 | 0 | 0.6 | 0.2 | 5 | 15 | 1.503 | 0.025 | 0.865 | 0.566 | 0.108 | 0.087 | 0.356 | 0.246 | 0.098 | 0.092 |
| 0.6 | 0 | 0 | 0 | 0.6 | 0 | 5 | 15 | 1.618 | 0.025 | 0.853 | 0.564 | 0.113 | 0.093 | 0.334 | 0.231 | 0.106 | 0.094 |
| 0.6 | 0 | 0 | 0 | 0 | 0 | 5 | 15 | 1.098 | 0.025 | 0.703 | 0.603 | 0.078 | 0.070 | 0.232 | 0.183 | 0.122 | 0.097 |
| 0 | 0 | 0.6 | 0.2 | 0.6 | 0.2 | 5 | 15 | 1.300 | 0.027 | 0.694 | 0.716 | 0.158 | 0.088 | 0.160 | 0.146 | 0.095 | 0.078 |
| 0 | 0 | 0.6 | 0.2 | 0.6 | 0 | 5 | 15 | 1.405 | 0.028 | 0.697 | 0.729 | 0.169 | 0.109 | 0.153 | 0.143 | 0.109 | 0.083 |
| 0 | 0 | 0.6 | 0.2 | 0 | 0 | 5 | 15 | 1.233 | 0.025 | 0.719 | 0.564 | 0.133 | 0.112 | 0.130 | 0.128 | 0.133 | 0.116 |
| 0 | 0 | 0.6 | 0 | 0.6 | 0.2 | 5 | 15 | 1.107 | 0.027 | 0.715 | 0.719 | 0.159 | 0.087 | 0.207 | 0.161 | 0.096 | 0.077 |
| 0 | 0 | 0.6 | 0 | 0.6 | 0 | 5 | 15 | 1.218 | 0.028 | 0.696 | 0.735 | 0.169 | 0.108 | 0.184 | 0.151 | 0.109 | 0.083 |
| 0 | 0 | 0.6 | 0 | 0 | 0 | 5 | 15 | 1.027 | 0.025 | 0.718 | 0.566 | 0.134 | 0.115 | 0.139 | 0.129 | 0.133 | 0.119 |
| 0 | 0 | 0 | 0 | 0.6 | 0.2 | 5 | 15 | 0.989 | 0.026 | 0.860 | 0.709 | 0.149 | 0.084 | 0.353 | 0.244 | 0.089 | 0.074 |
| 0 | 0 | 0 | 0 | 0.6 | 0 | 5 | 15 | 1.060 | 0.027 | 0.842 | 0.720 | 0.154 | 0.104 | 0.332 | 0.234 | 0.097 | 0.078 |
| 0 | 0 | 0 | 0 | 0 | 0 | 5 | 15 | 0.883 | 0.025 | 0.705 | 0.559 | 0.125 | 0.112 | 0.233 | 0.193 | 0.124 | 0.116 |

Table 13: RMSE of Parameters, $N = 100$, $T = 5$, $\theta = 0.5$

| **Configuration** | | | | | | | | **RMSE** | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\rho_1^L$ | $\rho_2^L$ | $\rho_1^A$ | $\rho_2^A$ | $\rho_1^B$ | $\rho_2^B$ | $\sigma_\mu^2$ | $\sigma_\nu^2$ | $\beta_0$ | $\beta_1$ | $\sigma_\mu^2$ | $\sigma_\nu^2$ | $\rho_1^L$ | $\rho_2^L$ | $\rho_1^A$ | $\rho_2^A$ | $\rho_1^B$ | $\rho_2^B$ |
| 0.6 | 0.2 | 0.6 | 0.2 | 0.6 | 0.2 | 10 | 10 | 2.442 | 0.043 | 1.789 | 0.819 | 0.162 | 0.139 | 0.283 | 0.211 | 0.193 | 0.156 |
| 0.6 | 0.2 | 0.6 | 0.2 | 0.6 | 0 | 10 | 10 | 2.983 | 0.043 | 1.767 | 0.831 | 0.157 | 0.144 | 0.245 | 0.208 | 0.202 | 0.153 |
| 0.6 | 0.2 | 0.6 | 0.2 | 0 | 0 | 10 | 10 | 2.121 | 0.046 | 1.811 | 0.885 | 0.134 | 0.124 | 0.199 | 0.189 | 0.233 | 0.155 |
| 0.6 | 0.2 | 0.6 | 0 | 0.6 | 0.2 | 10 | 10 | 2.859 | 0.043 | 1.843 | 0.819 | 0.162 | 0.138 | 0.331 | 0.217 | 0.188 | 0.156 |
| 0.6 | 0.2 | 0.6 | 0 | 0.6 | 0 | 10 | 10 | 3.967 | 0.044 | 1.806 | 0.842 | 0.171 | 0.148 | 0.297 | 0.207 | 0.203 | 0.156 |
| 0.6 | 0.2 | 0.6 | 0 | 0 | 0 | 10 | 10 | 1.785 | 0.046 | 1.790 | 0.883 | 0.127 | 0.117 | 0.218 | 0.179 | 0.228 | 0.149 |
| 0.6 | 0.2 | 0 | 0 | 0.6 | 0.2 | 10 | 10 | 2.904 | 0.044 | 2.101 | 0.821 | 0.161 | 0.139 | 0.406 | 0.289 | 0.178 | 0.154 |
| 0.6 | 0.2 | 0 | 0 | 0.6 | 0 | 10 | 10 | 4.928 | 0.045 | 2.091 | 0.836 | 0.181 | 0.155 | 0.404 | 0.281 | 0.185 | 0.156 |
| 0.6 | 0.2 | 0 | 0 | 0 | 0 | 10 | 10 | 1.633 | 0.046 | 1.868 | 0.860 | 0.126 | 0.119 | 0.323 | 0.241 | 0.217 | 0.150 |
| 0.6 | 0 | 0.6 | 0.2 | 0.6 | 0.2 | 10 | 10 | 2.424 | 0.045 | 1.810 | 0.823 | 0.186 | 0.134 | 0.287 | 0.223 | 0.195 | 0.156 |
| 0.6 | 0 | 0.6 | 0.2 | 0.6 | 0 | 10 | 10 | 2.278 | 0.044 | 1.792 | 0.841 | 0.180 | 0.141 | 0.250 | 0.215 | 0.210 | 0.160 |
| 0.6 | 0 | 0.6 | 0.2 | 0 | 0 | 10 | 10 | 2.077 | 0.045 | 1.806 | 0.884 | 0.152 | 0.134 | 0.197 | 0.193 | 0.240 | 0.182 |
| 0.6 | 0 | 0.6 | 0 | 0.6 | 0.2 | 10 | 10 | 2.236 | 0.045 | 1.845 | 0.814 | 0.179 | 0.129 | 0.333 | 0.223 | 0.186 | 0.152 |
| 0.6 | 0 | 0.6 | 0 | 0.6 | 0 | 10 | 10 | 2.174 | 0.044 | 1.804 | 0.830 | 0.172 | 0.138 | 0.300 | 0.213 | 0.204 | 0.158 |
| 0.6 | 0 | 0.6 | 0 | 0 | 0 | 10 | 10 | 1.868 | 0.045 | 1.778 | 0.888 | 0.141 | 0.130 | 0.216 | 0.184 | 0.234 | 0.182 |
| 0.6 | 0 | 0 | 0 | 0.6 | 0.2 | 10 | 10 | 2.212 | 0.045 | 2.079 | 0.815 | 0.174 | 0.136 | 0.404 | 0.291 | 0.172 | 0.152 |
| 0.6 | 0 | 0 | 0 | 0.6 | 0 | 10 | 10 | 2.116 | 0.045 | 2.112 | 0.817 | 0.165 | 0.140 | 0.403 | 0.278 | 0.183 | 0.154 |
| 0.6 | 0 | 0 | 0 | 0 | 0 | 10 | 10 | 1.638 | 0.046 | 1.838 | 0.864 | 0.138 | 0.131 | 0.324 | 0.252 | 0.221 | 0.178 |
| 0 | 0 | 0.6 | 0.2 | 0.6 | 0.2 | 10 | 10 | 2.338 | 0.047 | 1.837 | 0.988 | 0.295 | 0.161 | 0.299 | 0.223 | 0.202 | 0.148 |
| 0 | 0 | 0.6 | 0.2 | 0.6 | 0 | 10 | 10 | 2.226 | 0.047 | 1.834 | 0.977 | 0.286 | 0.178 | 0.260 | 0.215 | 0.220 | 0.154 |
| 0 | 0 | 0.6 | 0.2 | 0 | 0 | 10 | 10 | 2.021 | 0.046 | 1.847 | 0.883 | 0.233 | 0.179 | 0.203 | 0.197 | 0.251 | 0.200 |
| 0 | 0 | 0.6 | 0 | 0.6 | 0.2 | 10 | 10 | 2.067 | 0.048 | 1.866 | 0.989 | 0.289 | 0.157 | 0.342 | 0.225 | 0.193 | 0.145 |
| 0 | 0 | 0.6 | 0 | 0.6 | 0 | 10 | 10 | 2.037 | 0.047 | 1.817 | 0.989 | 0.289 | 0.177 | 0.307 | 0.214 | 0.214 | 0.153 |
| 0 | 0 | 0.6 | 0 | 0 | 0 | 10 | 10 | 1.746 | 0.046 | 1.845 | 0.880 | 0.237 | 0.183 | 0.227 | 0.192 | 0.250 | 0.201 |
| 0 | 0 | 0 | 0 | 0.6 | 0.2 | 10 | 10 | 1.884 | 0.048 | 2.047 | 1.001 | 0.276 | 0.157 | 0.409 | 0.291 | 0.173 | 0.144 |
| 0 | 0 | 0 | 0 | 0.6 | 0 | 10 | 10 | 1.875 | 0.047 | 2.051 | 0.999 | 0.275 | 0.175 | 0.404 | 0.277 | 0.188 | 0.147 |
| 0 | 0 | 0 | 0 | 0 | 0 | 10 | 10 | 1.660 | 0.046 | 1.837 | 0.877 | 0.236 | 0.190 | 0.335 | 0.263 | 0.240 | 0.203 |

Table 14: RMSE of Parameters, $N = 100$, $T = 10$, $\theta = 0.5$

| **Configuration** | | | | | | | | **RMSE** | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\rho_1^L$ | $\rho_2^L$ | $\rho_1^A$ | $\rho_2^A$ | $\rho_1^B$ | $\rho_2^B$ | $\sigma_\mu^2$ | $\sigma_\nu^2$ | $\beta_0$ | $\beta_1$ | $\sigma_\mu^2$ | $\sigma_\nu^2$ | $\rho_1^L$ | $\rho_2^L$ | $\rho_1^A$ | $\rho_2^A$ | $\rho_1^B$ | $\rho_2^B$ |
| 0.6 | 0.2 | 0.6 | 0.2 | 0.6 | 0.2 | 10 | 10 | 2.318 | 0.032 | 1.581 | 0.533 | 0.127 | 0.111 | 0.215 | 0.182 | 0.144 | 0.120 |
| 0.6 | 0.2 | 0.6 | 0.2 | 0.6 | 0 | 10 | 10 | 2.164 | 0.032 | 1.596 | 0.533 | 0.122 | 0.111 | 0.198 | 0.178 | 0.146 | 0.113 |
| 0.6 | 0.2 | 0.6 | 0.2 | 0 | 0 | 10 | 10 | 1.891 | 0.033 | 1.583 | 0.572 | 0.097 | 0.088 | 0.166 | 0.162 | 0.164 | 0.108 |
| 0.6 | 0.2 | 0.6 | 0 | 0.6 | 0.2 | 10 | 10 | 2.237 | 0.032 | 1.634 | 0.531 | 0.127 | 0.111 | 0.252 | 0.188 | 0.140 | 0.120 |
| 0.6 | 0.2 | 0.6 | 0 | 0.6 | 0 | 10 | 10 | 1.938 | 0.032 | 1.617 | 0.532 | 0.120 | 0.112 | 0.230 | 0.182 | 0.146 | 0.115 |
| 0.6 | 0.2 | 0.6 | 0 | 0 | 0 | 10 | 10 | 1.513 | 0.033 | 1.589 | 0.572 | 0.095 | 0.086 | 0.180 | 0.159 | 0.163 | 0.108 |
| 0.6 | 0.2 | 0 | 0 | 0.6 | 0.2 | 10 | 10 | 2.193 | 0.032 | 1.876 | 0.521 | 0.118 | 0.110 | 0.365 | 0.251 | 0.129 | 0.118 |
| 0.6 | 0.2 | 0 | 0 | 0.6 | 0 | 10 | 10 | 2.445 | 0.032 | 1.816 | 0.530 | 0.118 | 0.108 | 0.345 | 0.237 | 0.135 | 0.112 |
| 0.6 | 0.2 | 0 | 0 | 0 | 0 | 10 | 10 | 1.346 | 0.033 | 1.633 | 0.568 | 0.093 | 0.084 | 0.278 | 0.214 | 0.159 | 0.107 |
| 0.6 | 0 | 0.6 | 0.2 | 0.6 | 0.2 | 10 | 10 | 2.357 | 0.032 | 1.577 | 0.541 | 0.148 | 0.111 | 0.217 | 0.194 | 0.143 | 0.123 |
| 0.6 | 0 | 0.6 | 0.2 | 0.6 | 0 | 10 | 10 | 2.162 | 0.032 | 1.588 | 0.550 | 0.143 | 0.119 | 0.203 | 0.188 | 0.154 | 0.128 |
| 0.6 | 0 | 0.6 | 0.2 | 0 | 0 | 10 | 10 | 1.872 | 0.033 | 1.601 | 0.575 | 0.107 | 0.095 | 0.166 | 0.168 | 0.169 | 0.129 |
| 0.6 | 0 | 0.6 | 0 | 0.6 | 0.2 | 10 | 10 | 2.039 | 0.032 | 1.610 | 0.538 | 0.143 | 0.109 | 0.255 | 0.197 | 0.139 | 0.120 |
| 0.6 | 0 | 0.6 | 0 | 0.6 | 0 | 10 | 10 | 2.027 | 0.032 | 1.596 | 0.551 | 0.144 | 0.119 | 0.237 | 0.189 | 0.154 | 0.126 |
| 0.6 | 0 | 0.6 | 0 | 0 | 0 | 10 | 10 | 1.563 | 0.033 | 1.594 | 0.579 | 0.106 | 0.095 | 0.180 | 0.165 | 0.168 | 0.131 |
| 0.6 | 0 | 0 | 0 | 0.6 | 0.2 | 10 | 10 | 1.883 | 0.032 | 1.859 | 0.528 | 0.135 | 0.108 | 0.366 | 0.254 | 0.126 | 0.117 |
| 0.6 | 0 | 0 | 0 | 0.6 | 0 | 10 | 10 | 1.911 | 0.032 | 1.829 | 0.534 | 0.134 | 0.117 | 0.355 | 0.248 | 0.138 | 0.123 |
| 0.6 | 0 | 0 | 0 | 0 | 0 | 10 | 10 | 1.376 | 0.033 | 1.641 | 0.569 | 0.104 | 0.094 | 0.280 | 0.222 | 0.164 | 0.129 |
| 0 | 0 | 0.6 | 0.2 | 0.6 | 0.2 | 10 | 10 | 2.074 | 0.035 | 1.606 | 0.627 | 0.214 | 0.127 | 0.213 | 0.182 | 0.136 | 0.109 |
| 0 | 0 | 0.6 | 0.2 | 0.6 | 0 | 10 | 10 | 2.069 | 0.034 | 1.610 | 0.633 | 0.215 | 0.148 | 0.203 | 0.183 | 0.152 | 0.119 |
| 0 | 0 | 0.6 | 0.2 | 0 | 0 | 10 | 10 | 1.858 | 0.033 | 1.622 | 0.542 | 0.163 | 0.144 | 0.166 | 0.172 | 0.171 | 0.152 |
| 0 | 0 | 0.6 | 0 | 0.6 | 0.2 | 10 | 10 | 1.684 | 0.035 | 1.636 | 0.637 | 0.215 | 0.127 | 0.257 | 0.195 | 0.136 | 0.109 |
| 0 | 0 | 0.6 | 0 | 0.6 | 0 | 10 | 10 | 1.724 | 0.035 | 1.613 | 0.645 | 0.217 | 0.147 | 0.239 | 0.190 | 0.149 | 0.117 |
| 0 | 0 | 0.6 | 0 | 0 | 0 | 10 | 10 | 1.426 | 0.033 | 1.632 | 0.541 | 0.168 | 0.148 | 0.184 | 0.172 | 0.172 | 0.154 |
| 0 | 0 | 0 | 0 | 0.6 | 0.2 | 10 | 10 | 1.406 | 0.034 | 1.824 | 0.635 | 0.202 | 0.118 | 0.367 | 0.252 | 0.124 | 0.104 |
| 0 | 0 | 0 | 0 | 0.6 | 0 | 10 | 10 | 1.495 | 0.034 | 1.798 | 0.652 | 0.207 | 0.139 | 0.353 | 0.249 | 0.135 | 0.110 |
| 0 | 0 | 0 | 0 | 0 | 0 | 10 | 10 | 1.233 | 0.033 | 1.650 | 0.540 | 0.166 | 0.148 | 0.285 | 0.233 | 0.168 | 0.154 |

Table 15: RMSE of Parameters, $N = 200$, $T = 5$, $\theta = 0.5$

| **Configuration** | | | | | | | | **RMSE** | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\rho_1^L$ | $\rho_2^L$ | $\rho_1^A$ | $\rho_2^A$ | $\rho_1^B$ | $\rho_2^B$ | $\sigma_\mu^2$ | $\sigma_\nu^2$ | $\beta_0$ | $\beta_1$ | $\sigma_\mu^2$ | $\sigma_\nu^2$ | $\rho_1^L$ | $\rho_2^L$ | $\rho_1^A$ | $\rho_2^A$ | $\rho_1^B$ | $\rho_2^B$ |
| 0.6 | 0.2 | 0.6 | 0.2 | 0.6 | 0.2 | 10 | 10 | 2.185 | 0.030 | 1.279 | 0.548 | 0.119 | 0.103 | 0.177 | 0.157 | 0.132 | 0.114 |
| 0.6 | 0.2 | 0.6 | 0.2 | 0.6 | 0 | 10 | 10 | 1.930 | 0.031 | 1.291 | 0.550 | 0.112 | 0.101 | 0.164 | 0.153 | 0.134 | 0.108 |
| 0.6 | 0.2 | 0.6 | 0.2 | 0 | 0 | 10 | 10 | 1.647 | 0.031 | 1.299 | 0.597 | 0.093 | 0.086 | 0.137 | 0.136 | 0.160 | 0.113 |
| 0.6 | 0.2 | 0.6 | 0 | 0.6 | 0.2 | 10 | 10 | 2.122 | 0.031 | 1.306 | 0.544 | 0.117 | 0.107 | 0.200 | 0.162 | 0.130 | 0.117 |
| 0.6 | 0.2 | 0.6 | 0 | 0.6 | 0 | 10 | 10 | 1.840 | 0.031 | 1.294 | 0.539 | 0.111 | 0.105 | 0.182 | 0.155 | 0.133 | 0.112 |
| 0.6 | 0.2 | 0.6 | 0 | 0 | 0 | 10 | 10 | 1.407 | 0.031 | 1.302 | 0.594 | 0.092 | 0.085 | 0.143 | 0.131 | 0.158 | 0.112 |
| 0.6 | 0.2 | 0 | 0 | 0.6 | 0.2 | 10 | 10 | 2.079 | 0.031 | 1.549 | 0.540 | 0.107 | 0.102 | 0.321 | 0.227 | 0.118 | 0.112 |
| 0.6 | 0.2 | 0 | 0 | 0.6 | 0 | 10 | 10 | 2.351 | 0.031 | 1.526 | 0.540 | 0.106 | 0.104 | 0.307 | 0.218 | 0.121 | 0.111 |
| 0.6 | 0.2 | 0 | 0 | 0 | 0 | 10 | 10 | 1.282 | 0.031 | 1.312 | 0.584 | 0.088 | 0.081 | 0.223 | 0.173 | 0.152 | 0.110 |
| 0.6 | 0 | 0.6 | 0.2 | 0.6 | 0.2 | 10 | 10 | 2.151 | 0.030 | 1.284 | 0.554 | 0.139 | 0.107 | 0.184 | 0.164 | 0.133 | 0.121 |
| 0.6 | 0 | 0.6 | 0.2 | 0.6 | 0 | 10 | 10 | 1.997 | 0.031 | 1.271 | 0.563 | 0.137 | 0.111 | 0.170 | 0.159 | 0.144 | 0.123 |
| 0.6 | 0 | 0.6 | 0.2 | 0 | 0 | 10 | 10 | 1.628 | 0.031 | 1.298 | 0.592 | 0.106 | 0.093 | 0.137 | 0.138 | 0.165 | 0.130 |
| 0.6 | 0 | 0.6 | 0 | 0.6 | 0.2 | 10 | 10 | 1.995 | 0.030 | 1.303 | 0.553 | 0.137 | 0.105 | 0.209 | 0.169 | 0.132 | 0.118 |
| 0.6 | 0 | 0.6 | 0 | 0.6 | 0 | 10 | 10 | 1.910 | 0.031 | 1.264 | 0.553 | 0.134 | 0.114 | 0.191 | 0.162 | 0.142 | 0.123 |
| 0.6 | 0 | 0.6 | 0 | 0 | 0 | 10 | 10 | 1.488 | 0.031 | 1.294 | 0.594 | 0.107 | 0.095 | 0.146 | 0.136 | 0.166 | 0.134 |
| 0.6 | 0 | 0 | 0 | 0.6 | 0.2 | 10 | 10 | 1.874 | 0.030 | 1.569 | 0.550 | 0.128 | 0.104 | 0.327 | 0.238 | 0.120 | 0.115 |
| 0.6 | 0 | 0 | 0 | 0.6 | 0 | 10 | 10 | 1.784 | 0.030 | 1.541 | 0.546 | 0.120 | 0.111 | 0.312 | 0.228 | 0.125 | 0.118 |
| 0.6 | 0 | 0 | 0 | 0 | 0 | 10 | 10 | 1.317 | 0.031 | 1.313 | 0.581 | 0.099 | 0.091 | 0.226 | 0.184 | 0.157 | 0.130 |
| 0 | 0 | 0.6 | 0.2 | 0.6 | 0.2 | 10 | 10 | 1.819 | 0.033 | 1.306 | 0.665 | 0.209 | 0.120 | 0.176 | 0.152 | 0.126 | 0.102 |
| 0 | 0 | 0.6 | 0.2 | 0.6 | 0 | 10 | 10 | 1.838 | 0.033 | 1.314 | 0.662 | 0.211 | 0.138 | 0.164 | 0.152 | 0.141 | 0.112 |
| 0 | 0 | 0.6 | 0.2 | 0 | 0 | 10 | 10 | 1.582 | 0.031 | 1.333 | 0.559 | 0.164 | 0.133 | 0.138 | 0.141 | 0.167 | 0.147 |
| 0 | 0 | 0.6 | 0 | 0.6 | 0.2 | 10 | 10 | 1.574 | 0.033 | 1.334 | 0.671 | 0.209 | 0.121 | 0.207 | 0.162 | 0.125 | 0.102 |
| 0 | 0 | 0.6 | 0 | 0.6 | 0 | 10 | 10 | 1.626 | 0.033 | 1.324 | 0.676 | 0.214 | 0.142 | 0.190 | 0.159 | 0.140 | 0.112 |
| 0 | 0 | 0.6 | 0 | 0 | 0 | 10 | 10 | 1.313 | 0.031 | 1.343 | 0.563 | 0.167 | 0.141 | 0.147 | 0.144 | 0.168 | 0.152 |
| 0 | 0 | 0 | 0 | 0.6 | 0.2 | 10 | 10 | 1.395 | 0.032 | 1.551 | 0.664 | 0.195 | 0.118 | 0.331 | 0.237 | 0.116 | 0.098 |
| 0 | 0 | 0 | 0 | 0.6 | 0 | 10 | 10 | 1.427 | 0.032 | 1.526 | 0.669 | 0.197 | 0.138 | 0.312 | 0.232 | 0.127 | 0.107 |
| 0 | 0 | 0 | 0 | 0 | 0 | 10 | 10 | 1.173 | 0.031 | 1.313 | 0.563 | 0.161 | 0.141 | 0.232 | 0.203 | 0.161 | 0.150 |

Table 16: RMSE of Parameters, $N = 200$, $T = 10$, $\theta = 0.5$

| **Configuration** | | | | | | | | **RMSE** | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\rho_1^L$ | $\rho_2^L$ | $\rho_1^A$ | $\rho_2^A$ | $\rho_1^B$ | $\rho_2^B$ | $\sigma_\mu^2$ | $\sigma_\nu^2$ | $\beta_0$ | $\beta_1$ | $\sigma_\mu^2$ | $\sigma_\nu^2$ | $\rho_1^L$ | $\rho_2^L$ | $\rho_1^A$ | $\rho_2^A$ | $\rho_1^B$ | $\rho_2^B$ |
| 0.6 | 0.2 | 0.6 | 0.2 | 0.6 | 0.2 | 10 | 10 | 2.024 | 0.022 | 1.157 | 0.370 | 0.097 | 0.084 | 0.140 | 0.127 | 0.100 | 0.091 |
| 0.6 | 0.2 | 0.6 | 0.2 | 0.6 | 0 | 10 | 10 | 1.784 | 0.023 | 1.157 | 0.369 | 0.090 | 0.082 | 0.134 | 0.121 | 0.100 | 0.084 |
| 0.6 | 0.2 | 0.6 | 0.2 | 0 | 0 | 10 | 10 | 1.476 | 0.023 | 1.164 | 0.402 | 0.069 | 0.061 | 0.113 | 0.109 | 0.115 | 0.076 |
| 0.6 | 0.2 | 0.6 | 0 | 0.6 | 0.2 | 10 | 10 | 1.965 | 0.022 | 1.168 | 0.368 | 0.098 | 0.085 | 0.157 | 0.130 | 0.100 | 0.089 |
| 0.6 | 0.2 | 0.6 | 0 | 0.6 | 0 | 10 | 10 | 1.684 | 0.023 | 1.157 | 0.368 | 0.088 | 0.082 | 0.148 | 0.122 | 0.099 | 0.085 |
| 0.6 | 0.2 | 0.6 | 0 | 0 | 0 | 10 | 10 | 1.121 | 0.023 | 1.164 | 0.402 | 0.069 | 0.061 | 0.121 | 0.107 | 0.115 | 0.077 |
| 0.6 | 0.2 | 0 | 0 | 0.6 | 0.2 | 10 | 10 | 1.927 | 0.022 | 1.361 | 0.368 | 0.094 | 0.083 | 0.273 | 0.193 | 0.094 | 0.087 |
| 0.6 | 0.2 | 0 | 0 | 0.6 | 0 | 10 | 10 | 1.635 | 0.023 | 1.303 | 0.366 | 0.084 | 0.080 | 0.250 | 0.176 | 0.093 | 0.084 |
| 0.6 | 0.2 | 0 | 0 | 0 | 0 | 10 | 10 | 0.981 | 0.024 | 1.170 | 0.399 | 0.067 | 0.060 | 0.194 | 0.148 | 0.113 | 0.076 |
| 0.6 | 0 | 0.6 | 0.2 | 0.6 | 0.2 | 10 | 10 | 1.844 | 0.023 | 1.151 | 0.375 | 0.106 | 0.089 | 0.143 | 0.136 | 0.101 | 0.094 |
| 0.6 | 0 | 0.6 | 0.2 | 0.6 | 0 | 10 | 10 | 1.805 | 0.023 | 1.166 | 0.377 | 0.113 | 0.091 | 0.138 | 0.129 | 0.111 | 0.095 |
| 0.6 | 0 | 0.6 | 0.2 | 0 | 0 | 10 | 10 | 1.462 | 0.023 | 1.165 | 0.399 | 0.076 | 0.068 | 0.113 | 0.111 | 0.117 | 0.093 |
| 0.6 | 0 | 0.6 | 0 | 0.6 | 0.2 | 10 | 10 | 1.595 | 0.023 | 1.165 | 0.380 | 0.110 | 0.086 | 0.163 | 0.139 | 0.101 | 0.092 |
| 0.6 | 0 | 0.6 | 0 | 0.6 | 0 | 10 | 10 | 1.672 | 0.023 | 1.163 | 0.377 | 0.115 | 0.093 | 0.156 | 0.131 | 0.111 | 0.096 |
| 0.6 | 0 | 0.6 | 0 | 0 | 0 | 10 | 10 | 1.167 | 0.023 | 1.165 | 0.398 | 0.075 | 0.069 | 0.121 | 0.111 | 0.116 | 0.095 |
| 0.6 | 0 | 0 | 0 | 0.6 | 0.2 | 10 | 10 | 1.437 | 0.023 | 1.361 | 0.377 | 0.104 | 0.085 | 0.281 | 0.204 | 0.094 | 0.090 |
| 0.6 | 0 | 0 | 0 | 0.6 | 0 | 10 | 10 | 1.571 | 0.023 | 1.301 | 0.375 | 0.108 | 0.091 | 0.259 | 0.189 | 0.102 | 0.092 |
| 0.6 | 0 | 0 | 0 | 0 | 0 | 10 | 10 | 1.026 | 0.024 | 1.171 | 0.395 | 0.073 | 0.067 | 0.195 | 0.156 | 0.114 | 0.093 |
| 0 | 0 | 0.6 | 0.2 | 0.6 | 0.2 | 10 | 10 | 1.488 | 0.025 | 1.174 | 0.471 | 0.152 | 0.085 | 0.134 | 0.122 | 0.091 | 0.075 |
| 0 | 0 | 0.6 | 0.2 | 0.6 | 0 | 10 | 10 | 1.559 | 0.026 | 1.180 | 0.477 | 0.159 | 0.103 | 0.132 | 0.123 | 0.102 | 0.080 |
| 0 | 0 | 0.6 | 0.2 | 0 | 0 | 10 | 10 | 1.388 | 0.023 | 1.184 | 0.368 | 0.117 | 0.103 | 0.114 | 0.115 | 0.118 | 0.109 |
| 0 | 0 | 0.6 | 0 | 0.6 | 0.2 | 10 | 10 | 1.112 | 0.025 | 1.179 | 0.473 | 0.151 | 0.081 | 0.153 | 0.124 | 0.091 | 0.073 |
| 0 | 0 | 0.6 | 0 | 0.6 | 0 | 10 | 10 | 1.223 | 0.026 | 1.176 | 0.481 | 0.160 | 0.104 | 0.150 | 0.126 | 0.102 | 0.079 |
| 0 | 0 | 0.6 | 0 | 0 | 0 | 10 | 10 | 0.995 | 0.023 | 1.186 | 0.369 | 0.117 | 0.106 | 0.122 | 0.115 | 0.118 | 0.111 |
| 0 | 0 | 0 | 0 | 0.6 | 0.2 | 10 | 10 | 0.939 | 0.024 | 1.340 | 0.467 | 0.144 | 0.081 | 0.275 | 0.196 | 0.086 | 0.071 |
| 0 | 0 | 0 | 0 | 0.6 | 0 | 10 | 10 | 1.037 | 0.025 | 1.277 | 0.475 | 0.150 | 0.101 | 0.257 | 0.189 | 0.095 | 0.076 |
| 0 | 0 | 0 | 0 | 0 | 0 | 10 | 10 | 0.822 | 0.023 | 1.168 | 0.369 | 0.115 | 0.106 | 0.197 | 0.165 | 0.116 | 0.111 |

As the model is relatively rich and relatively high-dimensional in terms of parameters of interest, simulating the whole response surface of tests as in Baltagi, Egger, and Pfaffermayr (2013) would be very time consuming. Therefore, we simulate selected points of that surface in the following way. We present the results of the tests for five true parameter configurations, where each one of the hypotheses is true for exactly one configuration, and there is one configuration for which none of the null hypotheses is true. Moreover, as with the bias and RMSE, we present the corresponding test results for four configurations of the sample size in the combinations $N = \{100, 200\}$ and $T = \{5, 10\}$ and each of these combinations for one of two cases of the relative importance of the cross-sectional variation in the disturbances, $\theta = \{0.25, 0.50\}$.

We present the rejection rates for the corresponding LR tests in Table 17 for $\theta = 0.25$ and in Table 18 for $\theta = 0.5$. The two tables are organized identically in terms of their horizontal and vertical structure. The vertical structure corresponds to true-parameter-sample-size-configuration blocks, and the horizontal structure reflects the corresponding null hypotheses.

In each vertical block, the first parameter configuration does not match with anyone of the four null hypotheses, so the rejection rate is the power of this test (not size adjusted). $H_0^1$ is true in the second row (and first column) of each vertical block, $H_0^2$ is true in the third row (and second column) of each vertical block, $H_0^3$ is true in the fourth row (and third column) of each vertical block, and $H_0^4$ is true in the fifth row (and fourth column) of each vertical block, so the rejection rate is the size of these tests, which is expected to be close to the nominal test size of 0.05.

Indeed, the results indicate that the rejection rates are close to the nominal test size. In Table 17, for low heterogeneity $\theta = 0.25$, the LR test is slightly undersized for $N = 100$ and $T = 5$ ranging between 0.035 for $H_0^1$, 0.04 for $H_0^2$, 0.041 for $H_0^3$, and 0.05 for $H_0^4$. But this improves as $N$ or $T$ − increases. In Table 18, when we double the heterogeneity $\theta = 0.5$, the LR test size for $N = 100$ and $T = 5$ ranges between 0.04 for $H_0^1$, 0.027 for $H_0^2$, 0.047 for $H_0^3$, and 0.045 for $H_0^4$. So, $H_0^2 : \rho_2^A = 0$ which tests that the second order network effect in the individual time-invariant error is undersized and affected by this heterogeneity increase. Again, this improves as $T$ doubles to 10, where the empirical size for $H_0^2$ becomes 0.045. It also improves when $N$ doubles to 200, where the empirical size for $H_0^2$ becomes 0.044. Overall, we conclude that the proposed model can be recommended for both point estimation and testing having demonstrated this with limited Monte Carlo experiments for panel data of small to moderate size.

Table 17: LR Tests

| Configuration | | | | | | | | $H_0^1$: $\rho_2^L = 0$ | $H_0^2$: $\rho_2^A = 0$ | $H_0^3$: $\rho_2^B = 0$ | $H_0^4$: $\rho_2^L = \rho_2^A = \rho_2^B = 0$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $N = 100,\ T = 5,\ \theta = 0.25$ | | | | | | | | | | | |
| $\rho_1^L$ | $\rho_2^L$ | $\rho_1^A$ | $\rho_2^A$ | $\rho_1^B$ | $\rho_2^B$ | $\sigma_\mu^2$ | $\sigma_\nu^2$ | | | | |
| 0.6 | 0.2 | 0.6 | 0.2 | 0.6 | 0.2 | 5 | 15 | 0.212 | 0.037 | 0.232 | 0.942 |
| 0.6 | 0 | 0.6 | 0.2 | 0.6 | 0.2 | 5 | 15 | 0.035 | 0.029 | 0.144 | 0.290 |
| 0.6 | 0.2 | 0.6 | 0 | 0.6 | 0.2 | 5 | 15 | 0.200 | 0.040 | 0.252 | 0.935 |
| 0.6 | 0.2 | 0.6 | 0.2 | 0.6 | 0 | 5 | 15 | 0.086 | 0.081 | 0.041 | 0.569 |
| 0.6 | 0 | 0.6 | 0 | 0.6 | 0 | 5 | 15 | 0.038 | 0.039 | 0.037 | 0.050 |
| $N = 100,\ T = 10,\ \theta = 0.25$ | | | | | | | | | | | |
| $\rho_1^L$ | $\rho_2^L$ | $\rho_1^A$ | $\rho_2^A$ | $\rho_1^B$ | $\rho_2^B$ | $\sigma_\mu^2$ | $\sigma_\nu^2$ | | | | |
| 0.6 | 0.2 | 0.6 | 0.2 | 0.6 | 0.2 | 5 | 15 | 0.423 | 0.087 | 0.381 | 0.999 |
| 0.6 | 0 | 0.6 | 0.2 | 0.6 | 0.2 | 5 | 15 | 0.052 | 0.073 | 0.252 | 0.656 |
| 0.6 | 0.2 | 0.6 | 0 | 0.6 | 0.2 | 5 | 15 | 0.398 | 0.036 | 0.412 | 0.999 |
| 0.6 | 0.2 | 0.6 | 0.2 | 0.6 | 0 | 5 | 15 | 0.223 | 0.113 | 0.040 | 0.885 |
| 0.6 | 0 | 0.6 | 0 | 0.6 | 0 | 5 | 15 | 0.052 | 0.041 | 0.055 | 0.052 |
| $N = 200,\ T = 5,\ \theta = 0.25$ | | | | | | | | | | | |
| $\rho_1^L$ | $\rho_2^L$ | $\rho_1^A$ | $\rho_2^A$ | $\rho_1^B$ | $\rho_2^B$ | $\sigma_\mu^2$ | $\sigma_\nu^2$ | | | | |
| 0.6 | 0.2 | 0.6 | 0.2 | 0.6 | 0.2 | 5 | 15 | 0.518 | 0.127 | 0.390 | 0.999 |
| 0.6 | 0 | 0.6 | 0.2 | 0.6 | 0.2 | 5 | 15 | 0.041 | 0.104 | 0.270 | 0.671 |
| 0.6 | 0.2 | 0.6 | 0 | 0.6 | 0.2 | 5 | 15 | 0.454 | 0.042 | 0.396 | 0.999 |
| 0.6 | 0.2 | 0.6 | 0.2 | 0.6 | 0 | 5 | 15 | 0.243 | 0.185 | 0.040 | 0.956 |
| 0.6 | 0 | 0.6 | 0 | 0.6 | 0 | 5 | 15 | 0.041 | 0.053 | 0.045 | 0.042 |
| $N = 200,\ T = 10,\ \theta = 0.25$ | | | | | | | | | | | |
| $\rho_1^L$ | $\rho_2^L$ | $\rho_1^A$ | $\rho_2^A$ | $\rho_1^B$ | $\rho_2^B$ | $\sigma_\mu^2$ | $\sigma_\nu^2$ | | | | |
| 0.6 | 0.2 | 0.6 | 0.2 | 0.6 | 0.2 | 5 | 15 | 0.741 | 0.186 | 0.627 | 1.000 |
| 0.6 | 0 | 0.6 | 0.2 | 0.6 | 0.2 | 5 | 15 | 0.056 | 0.257 | 0.416 | 0.999 |
| 0.6 | 0.2 | 0.6 | 0 | 0.6 | 0.2 | 5 | 15 | 0.690 | 0.051 | 0.666 | 1.000 |
| 0.6 | 0.2 | 0.6 | 0.2 | 0.6 | 0 | 5 | 15 | 0.449 | 0.173 | 0.038 | 0.922 |
| 0.6 | 0 | 0.6 | 0 | 0.6 | 0 | 5 | 15 | 0.056 | 0.058 | 0.061 | 0.047 |

Table 18: LR Tests

| $\rho_1^L$ | $\rho_2^L$ | $\rho_1^A$ | $\rho_2^A$ | $\rho_1^B$ | $\rho_2^B$ | $\sigma_\mu^2$ | $\sigma_\nu^2$ | $H_0^1$: $\rho_2^L = 0$ | $H_0^2$: $\rho_2^A = 0$ | $H_0^3$: $\rho_2^B = 0$ | $H_0^4$: $\rho_2^L = \rho_2^A = \rho_2^B = 0$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Configuration** | | | | | | | | | | | |
| $N = 100$, $T = 5$, $\theta = 0.5$ | | | | | | | | | | | |
| 0.6 | 0.2 | 0.6 | 0.2 | 0.6 | 0.2 | 5 | 15 | 0.216 | 0.091 | 0.273 | 0.951 |
| 0.6 | 0 | 0.6 | 0.2 | 0.6 | 0.2 | 5 | 15 | 0.040 | 0.064 | 0.162 | 0.322 |
| 0.6 | 0.2 | 0.6 | 0 | 0.6 | 0.2 | 5 | 15 | 0.194 | 0.027 | 0.306 | 0.959 |
| 0.6 | 0.2 | 0.6 | 0.2 | 0.6 | 0 | 5 | 15 | 0.104 | 0.125 | 0.047 | 0.696 |
| 0.6 | 0 | 0.6 | 0 | 0.6 | 0 | 5 | 15 | 0.039 | 0.030 | 0.047 | 0.045 |
| $N = 100$, $T = 10$, $\theta = 0.5$ | | | | | | | | | | | |
| 0.6 | 0.2 | 0.6 | 0.2 | 0.6 | 0.2 | 5 | 15 | 0.449 | 0.119 | 0.397 | 1.000 |
| 0.6 | 0 | 0.6 | 0.2 | 0.6 | 0.2 | 5 | 15 | 0.047 | 0.103 | 0.275 | 0.674 |
| 0.6 | 0.2 | 0.6 | 0 | 0.6 | 0.2 | 5 | 15 | 0.407 | 0.045 | 0.448 | 1.000 |
| 0.6 | 0.2 | 0.6 | 0.2 | 0.6 | 0 | 5 | 15 | 0.263 | 0.147 | 0.052 | 0.922 |
| 0.6 | 0 | 0.6 | 0 | 0.6 | 0 | 5 | 15 | 0.051 | 0.047 | 0.047 | 0.052 |
| $N = 200$, $T = 5$, $\theta = 0.5$ | | | | | | | | | | | |
| 0.6 | 0.2 | 0.6 | 0.2 | 0.6 | 0.2 | 5 | 15 | 0.487 | 0.203 | 0.377 | 0.999 |
| 0.6 | 0 | 0.6 | 0.2 | 0.6 | 0.2 | 5 | 15 | 0.037 | 0.154 | 0.292 | 0.722 |
| 0.6 | 0.2 | 0.6 | 0 | 0.6 | 0.2 | 5 | 15 | 0.430 | 0.044 | 0.415 | 0.999 |
| 0.6 | 0.2 | 0.6 | 0.2 | 0.6 | 0 | 5 | 15 | 0.291 | 0.259 | 0.042 | 0.983 |
| 0.6 | 0 | 0.6 | 0 | 0.6 | 0 | 5 | 15 | 0.040 | 0.057 | 0.038 | 0.051 |
| $N = 200$, $T = 10$, $\theta = 0.5$ | | | | | | | | | | | |
| 0.6 | 0.2 | 0.6 | 0.2 | 0.6 | 0.2 | 5 | 15 | 0.763 | 0.241 | 0.650 | 1.000 |
| 0.6 | 0 | 0.6 | 0.2 | 0.6 | 0.2 | 5 | 15 | 0.058 | 0.323 | 0.439 | 0.999 |
| 0.6 | 0.2 | 0.6 | 0 | 0.6 | 0.2 | 5 | 15 | 0.711 | 0.044 | 0.694 | 1.000 |
| 0.6 | 0.2 | 0.6 | 0.2 | 0.6 | 0 | 5 | 15 | 0.522 | 0.207 | 0.040 | 0.936 |
| 0.6 | 0 | 0.6 | 0 | 0.6 | 0 | 5 | 15 | 0.056 | 0.054 | 0.061 | 0.057 |

## 4 Conclusions

This paper proposes a higher-order network panel-data model with a structure that permits discerning the influence of spatial and network interactions channeled through the presence of endogenous network lags of the dependent variable, exogenous network lags of the explanatory variables, and separate network lags in the time-invariant and the time-variant parts of the disturbances. All of the latter network links can be of the higher-order type and be simultaneously present in the model.

The availability of a model of this kind is desirable, because we often ex ante do not know either the exact decay function of network effects with an increasing network (or spatial) distance between the cross sectional units (or the nodes) in the network, nor may we know which ones of several potential channels of network links matter at all or matter more than others. Either lack of knowledge may come at the cost of bias or, at best, efficiency losses.

We found that the proposed model, which is a generalization of the one in Baltagi, Egger, and Pfaffermayr (2013), works remarkably well in terms of bias (and root-mean-squared error) already in modest sample sizes of $N = 100$ and $T = 5$. In practice, the networks typically consist of much more than just 100 cross-sectional units (or nodes). Hence, we are confident that this model can be recommended for applications with real-world data, where it may help unveiling the aforementioned desirable knowledge regarding decaying network links with greater network distance as well as the relative importance of different network channels at work.

# 5 References


**Badinger, H., Egger, P.H.,** 2011. Estimation of higher-order spatial autoregressive cross-section models with heteroskedastic disturbances. Papers in Regional Science. The Journal of the Regional Science Association International 90(1), 213–236.

———, 2013. Estimation and testing of higher-order spatial autoregressive panel data error component models. Journal of Geographical Systems 15(4), 453–489.

———, 2015. Fixed effects and random effects estimation of higher-order spatial autoregressive models with spatial autoregressive and heteroscedastic disturbances. Spatial Economic Analysis 10(1), 11–35.

**Bai, J., Baltagi, B.H., Pesaran, M.H.,** 2016. Cross-section dependence in panel data models: a special issue. Journal of Applied Econometrics 31(1), 1–3.

**Baltagi, B.H., Egger, P.H., Kesina, M.,** 2017. Determinants of firm-level domestic sales and exports with spillovers: Evidence from China. Journal of Econometrics 199(2), 184–201.

**Baltagi, B.H., Egger, P., Pfaffermayr, M.,** 2007. Estimating models of complex FDI: Are there third-country effects?. Journal of Econometrics 140(1), 260–281.

———, 2013. A generalized spatial panel data model with random effects. Econometric Reviews 32(5–6), 650–685.

**Baltagi, B.H., Pesaran, M.H.,** 2007. Heterogeneity and cross section dependence in panel data models: theory and applications introduction. Journal of Applied Econometrics 22(2), 229–232.

**Chudik, A., Pesaran, M.H.,** 2015. Large panel data models with cross-sectional dependence: a survey. In: Baltagi, B.H. (Ed.), The Oxford Handbook of Panel Data. Oxford University Press, New York, 3–45.

**Egger, P.H., Pfaffermayr, M., Winner, H.,** 2005. An unbalanced spatial panel data approach to US state tax competition. Economics Letters 88(3), 329–335.

**Ertur, C., Koch, W.,** 2007. Growth, technological interdependence and spatial externalities: Theory and evidence. Journal of Applied Econometrics 22(6), 1033–1062.

**Gupta, A., Robinson, P.M.,** 2015. Inference on higher-order spatial autoregressive models with increasingly many parameters. Journal of Econometrics 186(1), 19–31.

**Kelejian, H.H., Prucha, I.R.,** 2010. Specification and estimation of spatial autoregressive models with autoregressive and heteroskedastic disturbances. Journal of Econometrics 157(1), 53–67.

**Mundlak, Y.,** 1978. On the pooling of time series and cross section data. Econometrica 46(1), 69–85. (1978)

**Pinkse, J., Slade, M.E., Brett, C.,** 2002. Spatial price competition: A semiparametric approach. Econometrica 70(3), 1111–1153.

**Wansbeek, T., Kapteyn, A.,** 1982. A class of decompositions of the variance-covariance matrix of a generalized error components model. Econometrica 50(3), 713–724.